Benchmarking Using Value Efficiency Analysis

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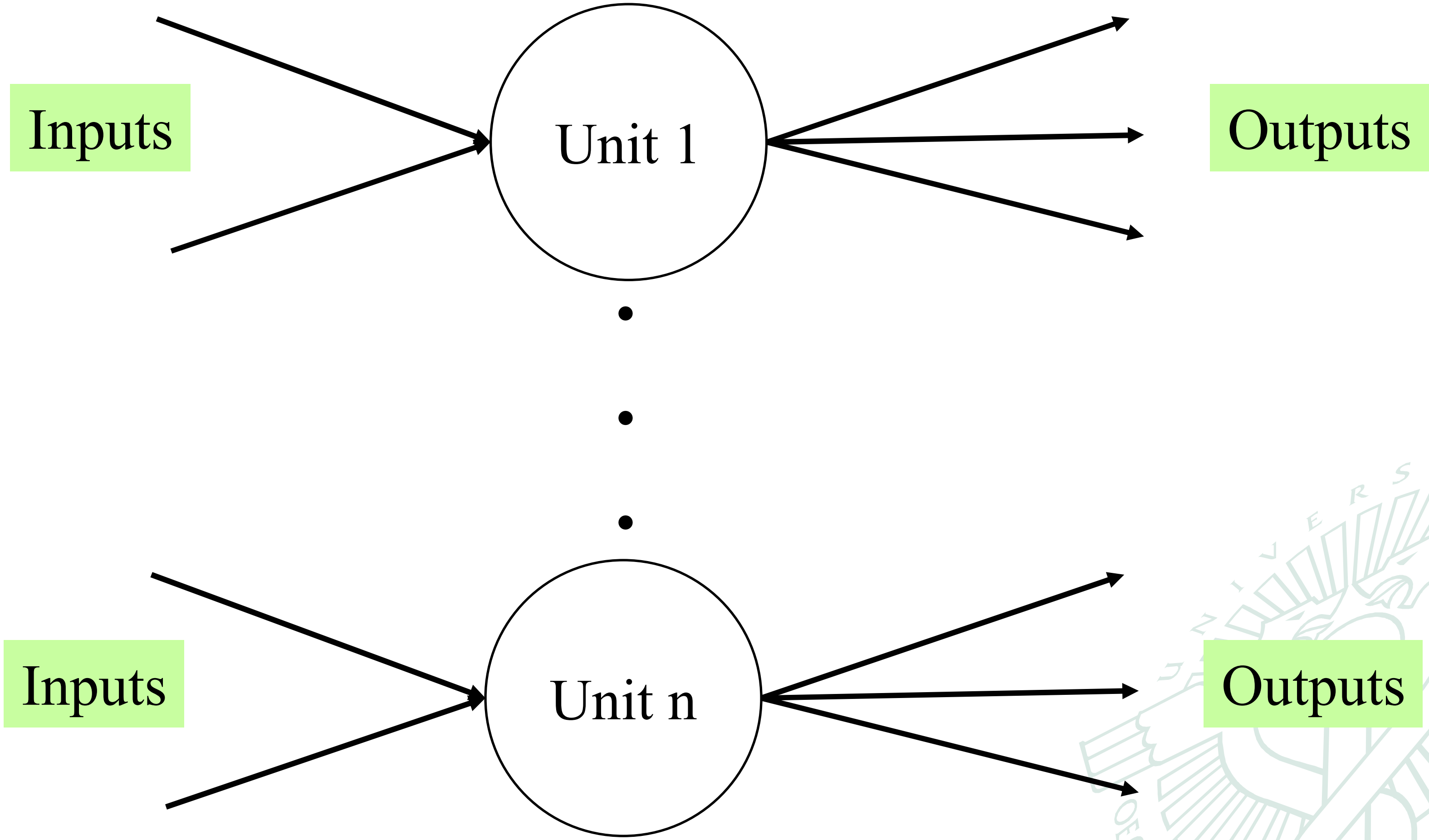
Problem ?

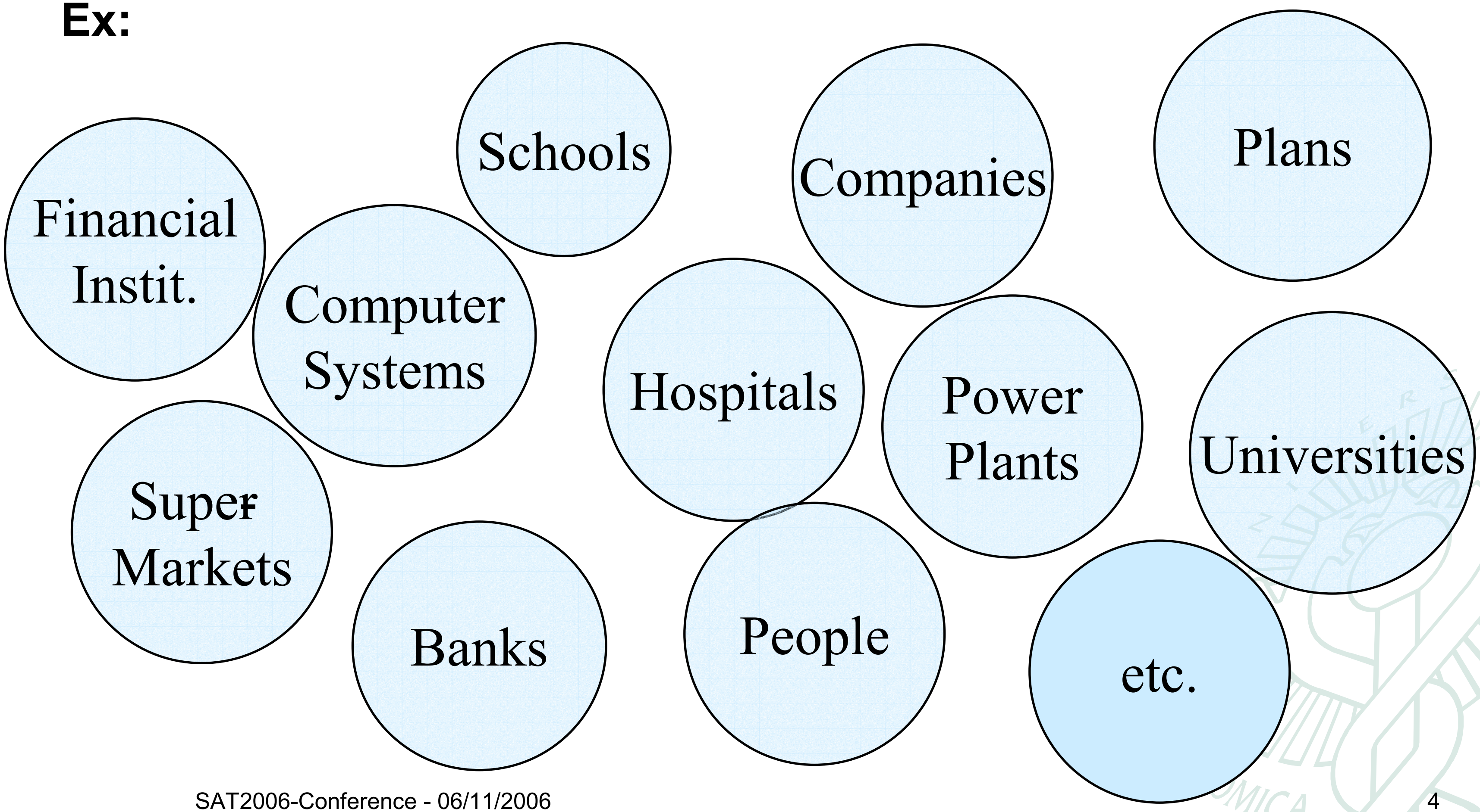
- How to compare different ”units” which are measured with several outputs and several inputs ?

Principle as a picture



Units
= Any systems having measurable inputs and outputs

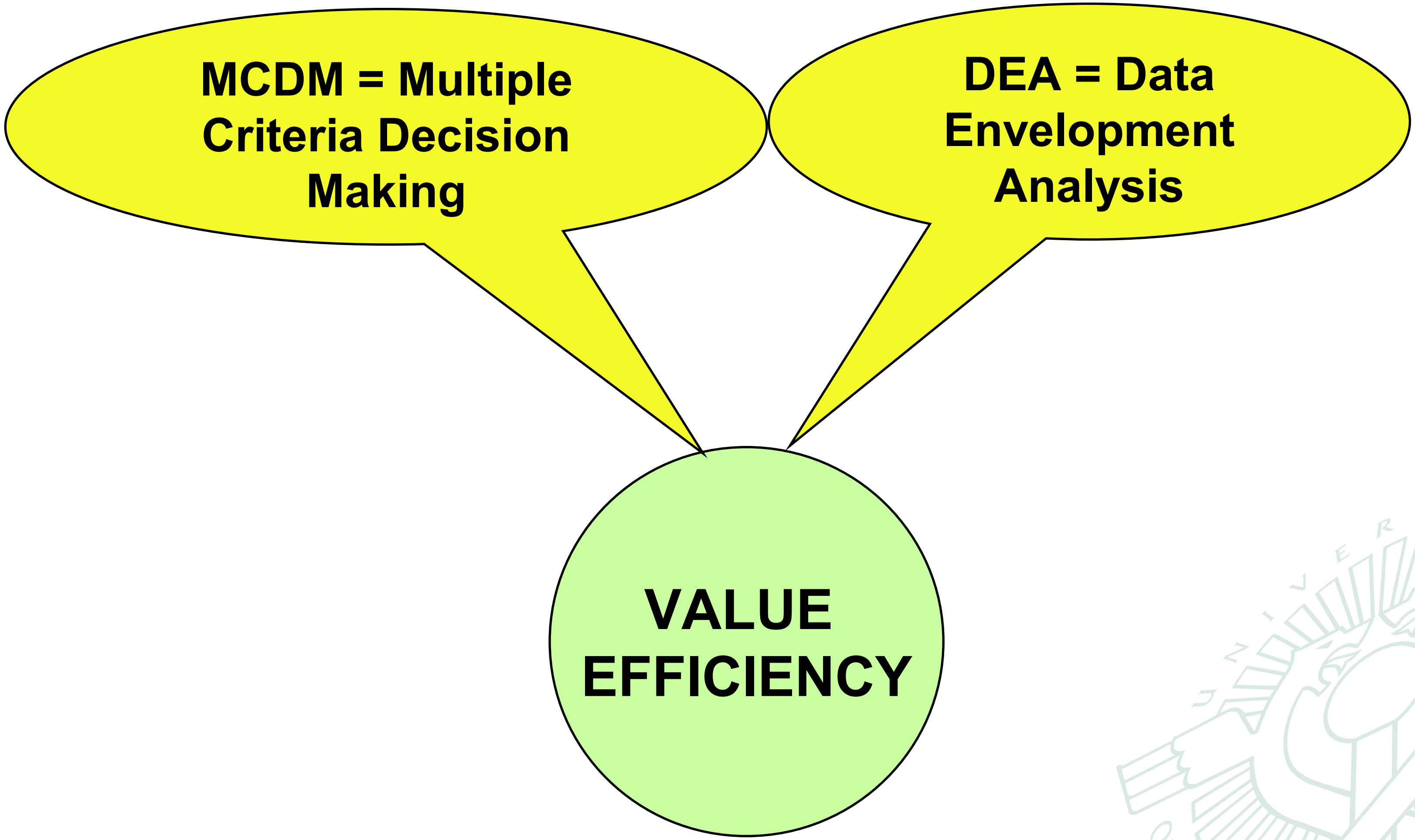
Ex:



Basic Idea

- Efficiency of the Units is carried out by Comparing Each Unit to Similar Units Using the Same Inputs and Outputs.
- ”Similar” means that the structure of the units is similar and they are operating in similar environments
- Each Unit is considered as a Black Box transforming Inputs into Outputs.
- Only Input- and Output - values are considered

What is needed ?



Basic Concepts in MCDM

MCDM = Multiple Criteria Decision Making

* Structuring and Solving Decision and Planning Problems Involving Multiple Criteria.

"Solving" =

DM will choose the "**Best**" alternative from among a set of available "**Reasonable"** ones

Best = The Most Preferred One

Reasonable = Efficient/Nondominated

Typology of MCDM-Problems

- **Multiple Criteria Evaluation Problems**
  - A finite number of Alternatives is explicitly known in the beginning of the solution process
- **Multiple Criteria Design Problems**
  - A number of alternatives is infinite and not countable
  - The alternatives are usually defined using a mathematical model formulation.
  - The alternatives are only implicitly known

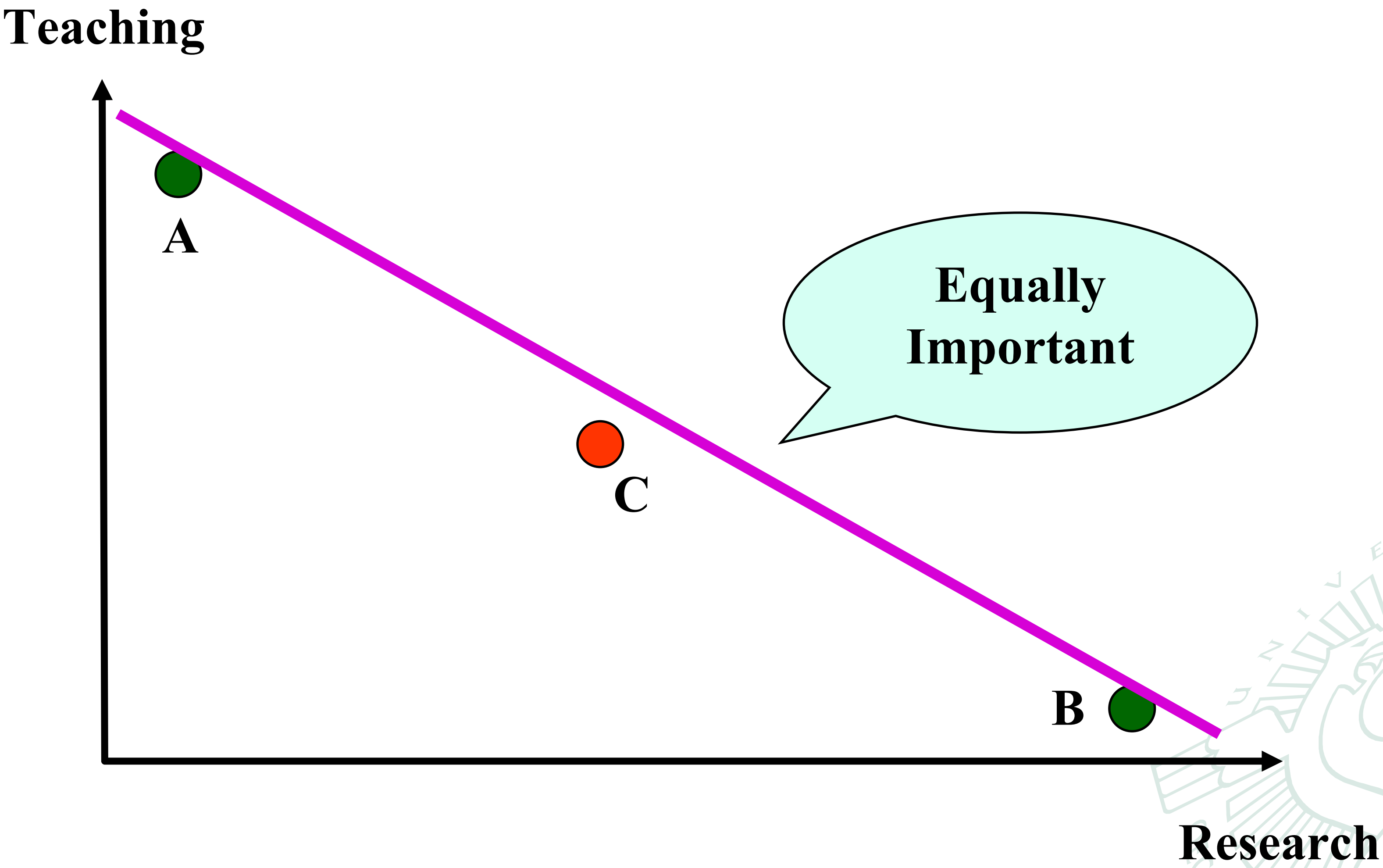
Example 1: Problems of Using Weights in Multiple Criteria Evaluation ?

Where to go to spend Holiday ?

	Price	Hiking	Fishing	Hunting	Swimming	Surfing
Places						
A	10	1	1	1	1	1
B	5	5	5	5	5	5
C	1	10	10	10	10	10

	Price	Hiking	Fishing	Hunting	Swimming	Surfing	
Weights	9	2	2	2	2	2	Σ
Places							
A							100
B							95
C	1	10	10	10	10	10	109

Example 2: Another Problem of Using Weights in Multiple Criteria Evaluation



Example 3: Investment (Multiple Criteria Design)
MOLP: Multiple Objective Linear Programming

- Invest 100,000 dollars profitably
- 4 possible investment options
- Invest in one or several options
- They are not riskless
- Returns (%) depend on the general state of the economy (Declining, Stable, Improving):

BASIC DATA

Economy\Options	1 (%)	2 (%)	3 (%)	4 (%)
Declining	-2	4	-7	15
Stable	5	3	9	4
Improving	3	0	10	-8

$0.5*opt_3 + 0.5*opt_4$

Economy\Options	1 (%)	2 (%)	3 (%)	4 (%)	X (%)
Declining	-2	4	-7	15	**4.0**
Stable	5	3	9	4	**6.5**
Improving	3	0	10	-8	**1.0**
Weights			**0.5**	**0.5**	

$(2/3)*opt_3 + (1/3)*opt_4$

Economy\Options	1 (%)	2 (%)	3 (%)	4 (%)	X (%)
Declining	-2	4	-7	15	**0.33**
Stable	5	3	9	4	**7.33**
Improving	3	0	10	-8	**4.00**
Weights			**2/3**	**1/3**	

DEA

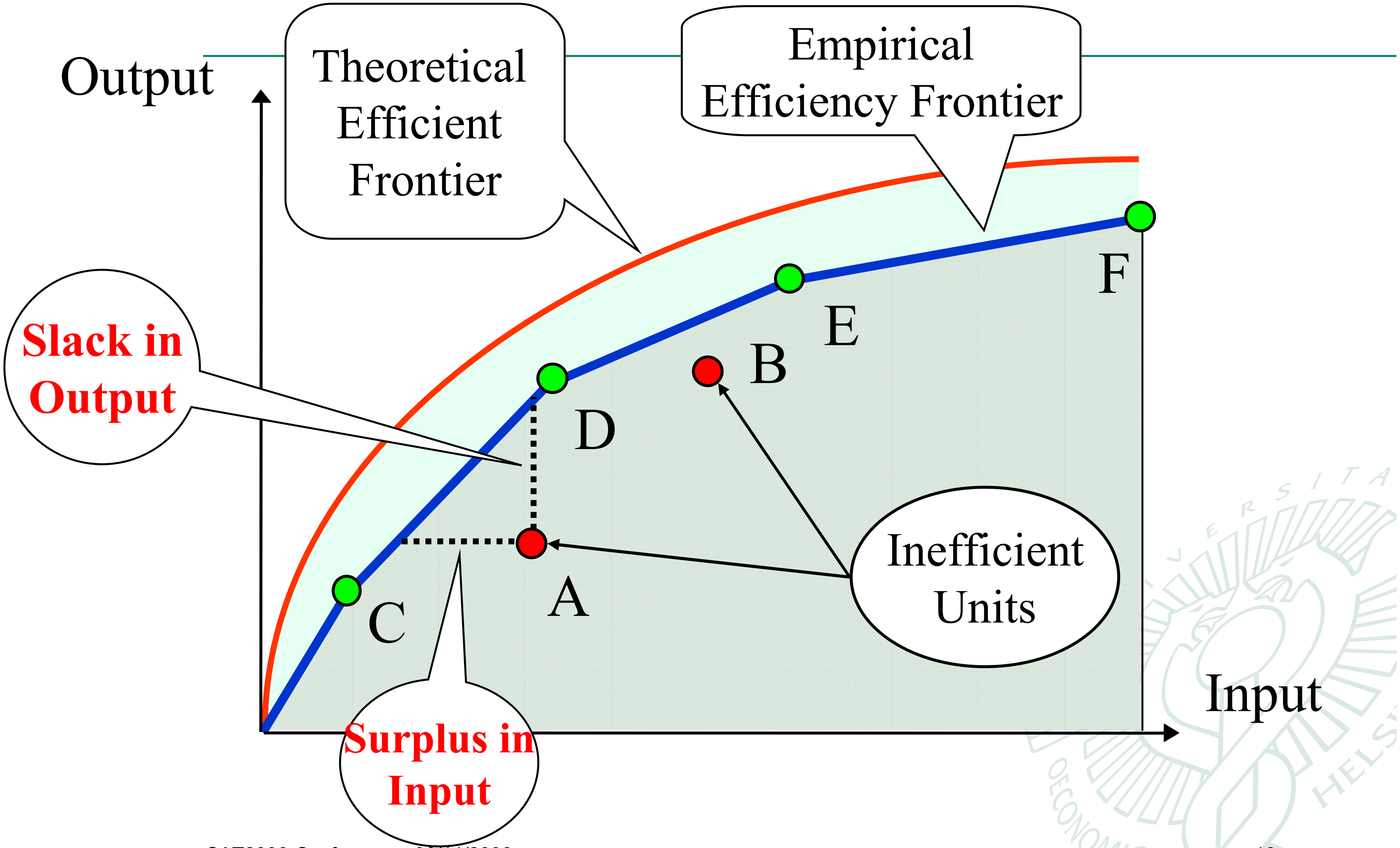
=

DATA

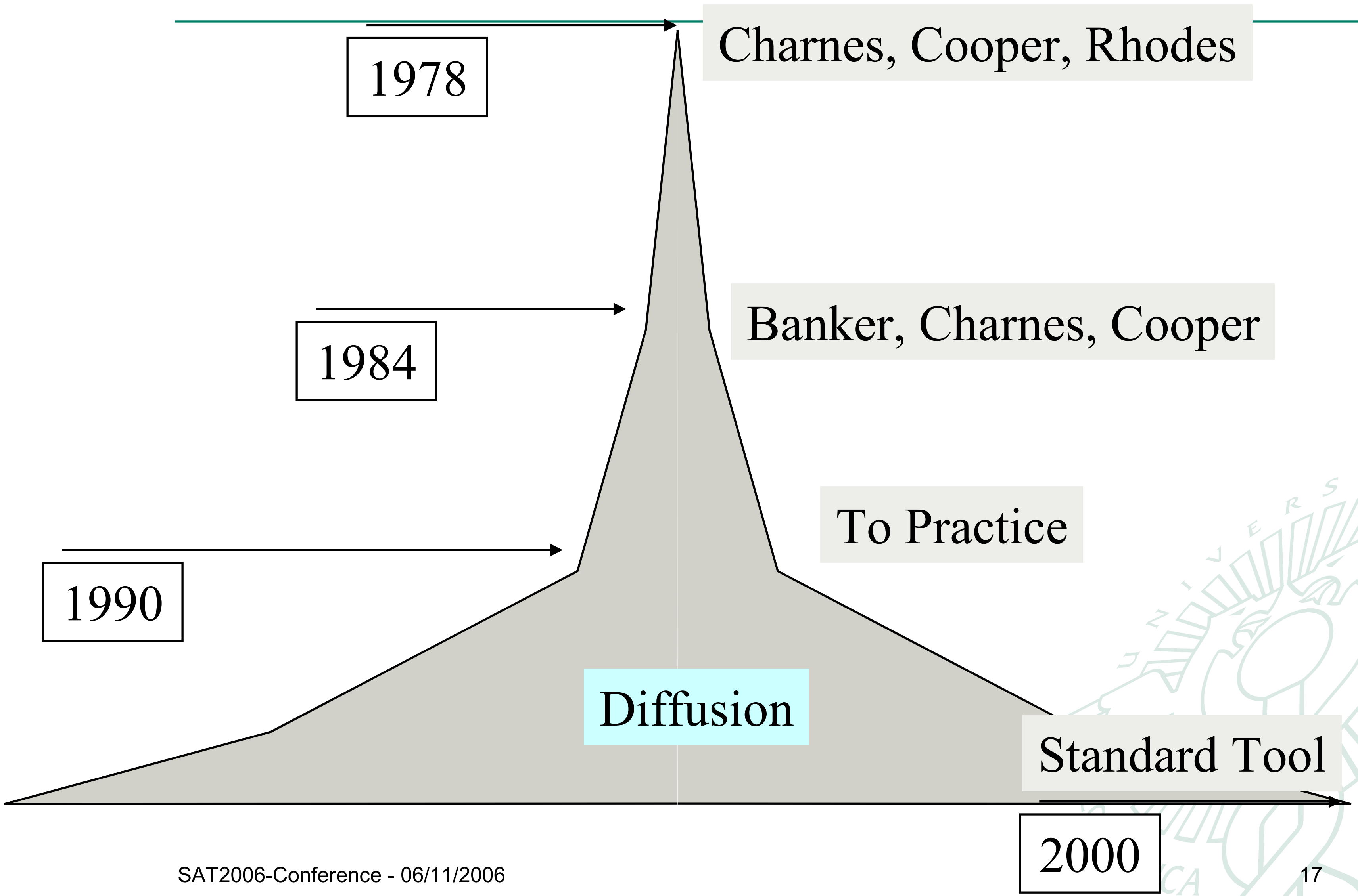
ENVELOPMENT

ANALYSIS

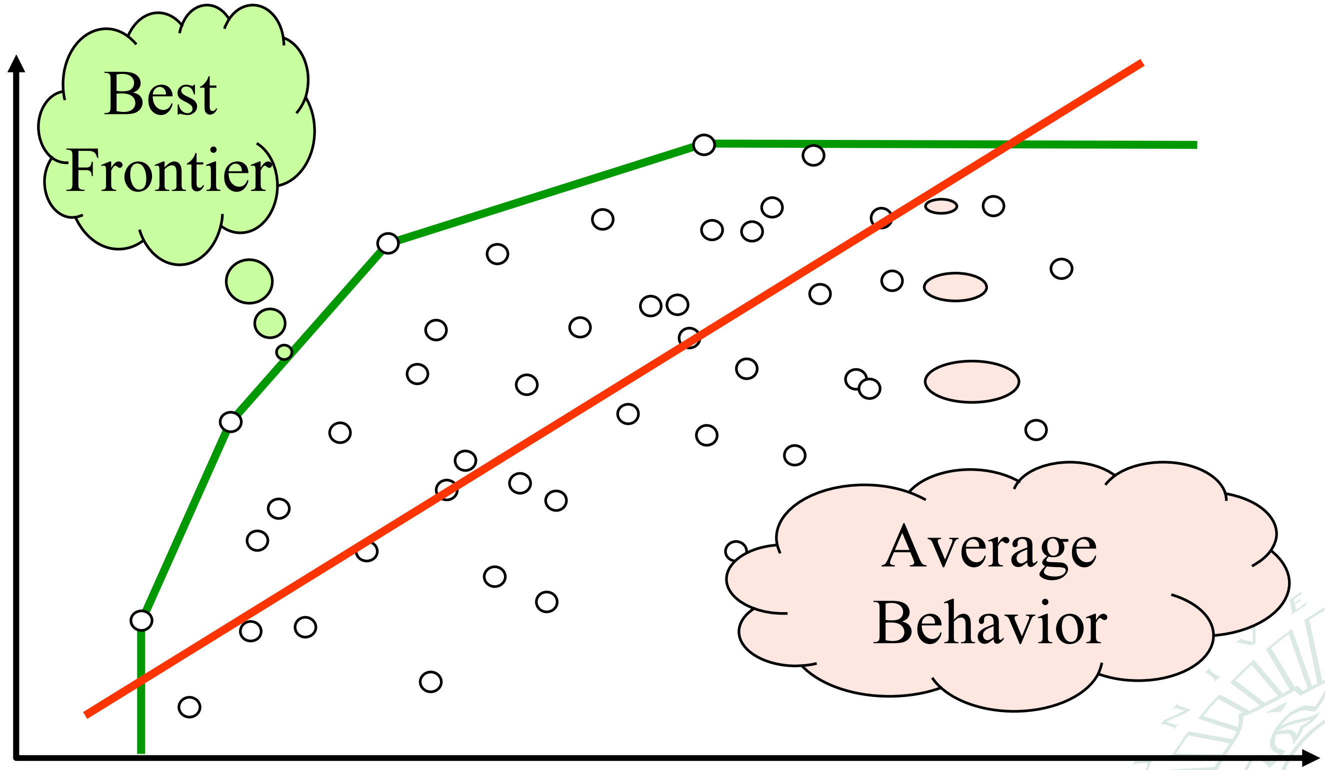
Illustration



Short History of DEA



DEA and Regression Analysis



How ?

Find the weights for all inputs and outputs in such a way that

Productivity

<u>**The Weighted Sum of Outputs**</u>

The Weighted Sum of Inputs

is as good as possible in relation to the other units.

HSE

Numerical Data

	A	B	C	D
Sales (milj. mk)	225	79	66	99
Profit (milj. mk)	5.0	0.2	1.5	1.9
Employees (10^3 t)	127	50	48	69
Floor Size (10^3 m^2)	8.1	2.5	2.3	3.0

Basic Model (Constant Returns to Scale)

$$\max \frac{225u_1 + 5u_2}{127v_1 + 8.1v_2}$$

Maximize the Productivity of Firm A

subject to:

$$\frac{225u_1 + 5u_2}{127v_1 + 8.1v_2} \leq 1, \quad \frac{79u_1 + 0.2u_2}{50v_1 + 2.5v_2} \leq 1,$$

$$\frac{66u_1 + 1.5u_2}{48v_1 + 2.3v_2} \leq 1, \quad \frac{99u_1 + 1.9u_2}{69v_1 + 3.0v_2} \leq$$

$$u_1, u_2, v_1, v_2 \geq 0$$

Provided that the Productivity of all firms are forced to be at most 1

What does DEA tell ?

- Identify a so-called Empirical Efficient Frontier
- Find Efficient Units on the frontier
- Provide Reference Units (Benchmarking Units) for Inefficient Units
- Produce Efficiency Score for Inefficient Units
- Propose Directions for Improvements

HSE

Basic Models

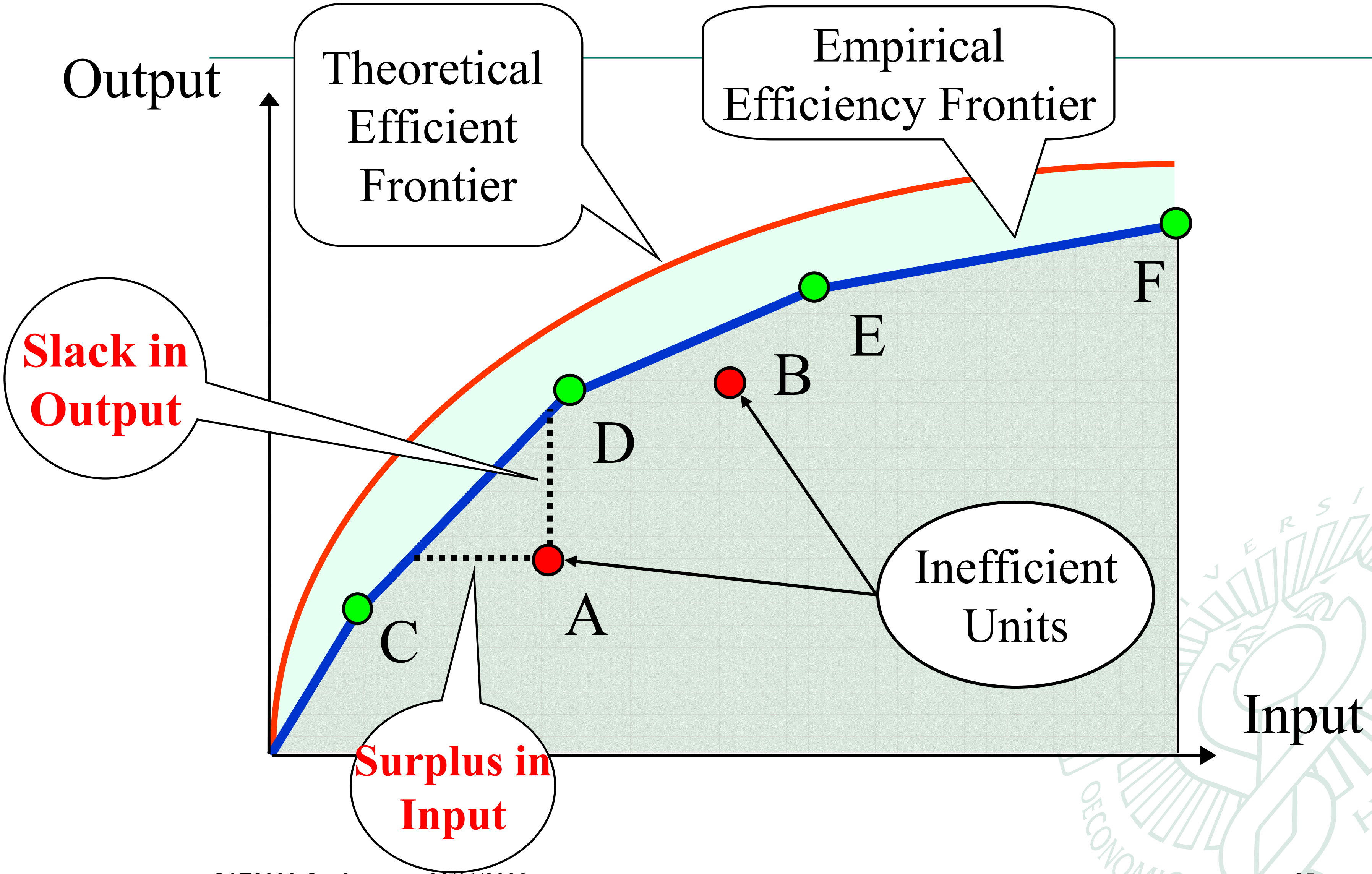
- CCR-model (**C**harnes, **C**ooper, **R**hodes)
  – Constant Returns to Scale
- BCC-model (**B**anker, **C**harnes, **C**ooper)
  – Variable Returns to Scale

Constant Returns to Scale Model

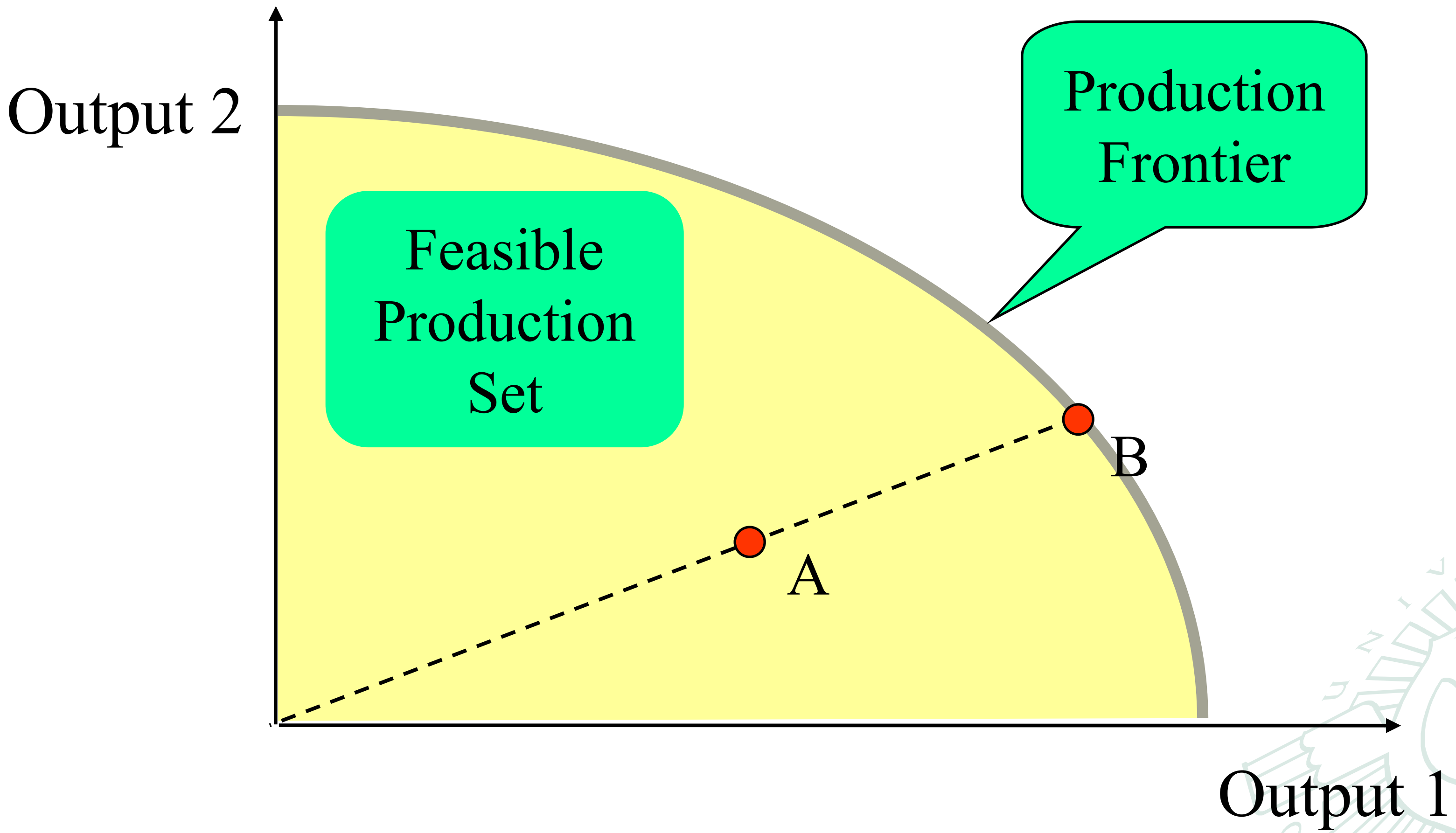
Summary Table of Output-Oriented CCR-Models:

Output-Oriented CCR Primal (E) (CCR_P - O)	Output-Oriented CCR Dual (D') (CCR_D - O)
$max\ Z_O = \theta + \varepsilon(\mathbf{1}^T \mathbf{s}^+ + \mathbf{1}^T \mathbf{s}^-)$ s.t. $\mathbf{Y}\lambda - \theta \mathbf{y}_0 - \mathbf{s}^+ = \mathbf{0}$ $\mathbf{X}\lambda + \mathbf{s}^- = \mathbf{x}_0$ $\lambda, \mathbf{s}^-, \mathbf{s}^+ \geq \mathbf{0}$ $\varepsilon > 0$	$min\ W_O = \boldsymbol{\nu}^T \mathbf{x}_0$ s.t. $\boldsymbol{\mu}^T \mathbf{y}_0 = 1$ $-\boldsymbol{\mu}^T \mathbf{Y} + \boldsymbol{\nu}^T \mathbf{X} \geq \mathbf{0}$ $\boldsymbol{\mu}, \boldsymbol{\nu} \geq \varepsilon \mathbf{1}$ $\varepsilon > 0$

Illustration



Technical Efficiency; 2 Outputs



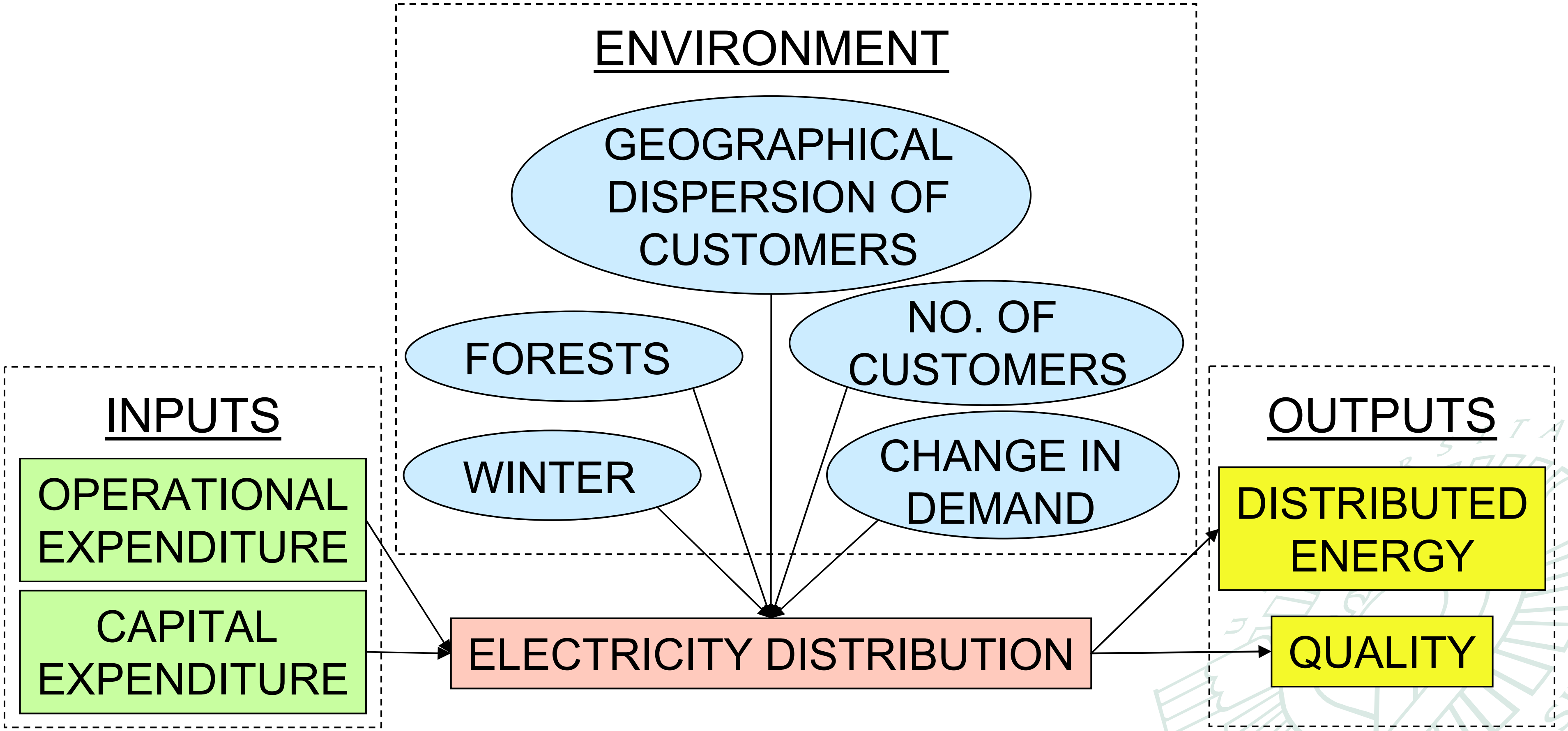
Summary

- Data consist of only inputs, outputs, and environmental factors of the units
- Applicable to evaluate (relative) technical efficiency of units
- Measure for Inefficiency
- Benchmarking Units for Inefficient Units
- Hints to improve efficiency
- It is possible to take into account preference information

Assessment of Cost Efficiency in Finnish Electricity Distribution (Pekka Korhonen & Mikko Syrjänen)

- The purpose was to develop an approach
  - for evaluation of cost efficiency
  - in Finnish electricity distribution
  - based on DEA
  - to be used by the Finnish regulator (Energy Market Authority, EMA)

Summary of factors



VALUE EFFICIENCY ANALYSIS

Halme, Joro, Korhonen, Salo, Wallenius [1998]

Efficient = Good ??

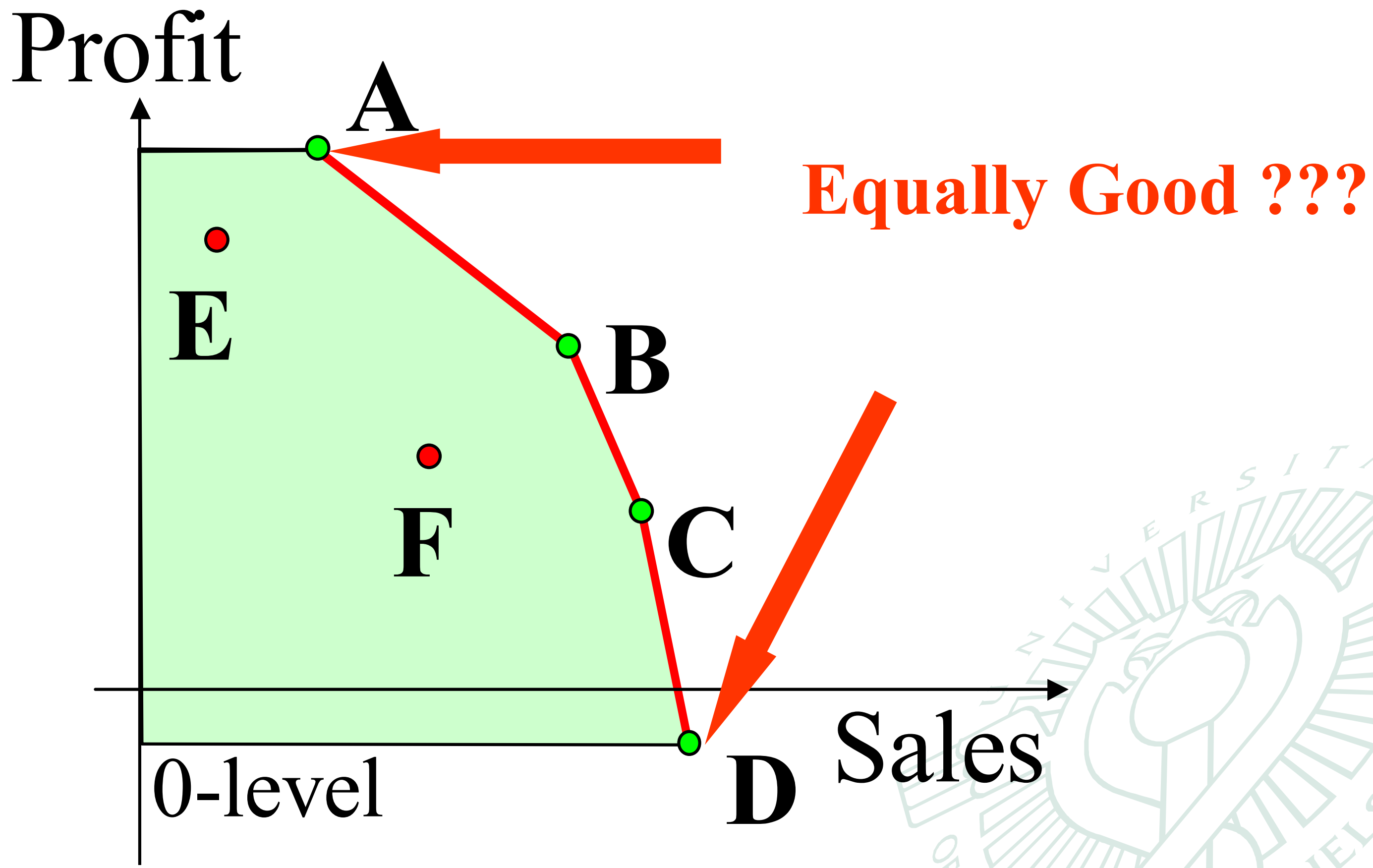
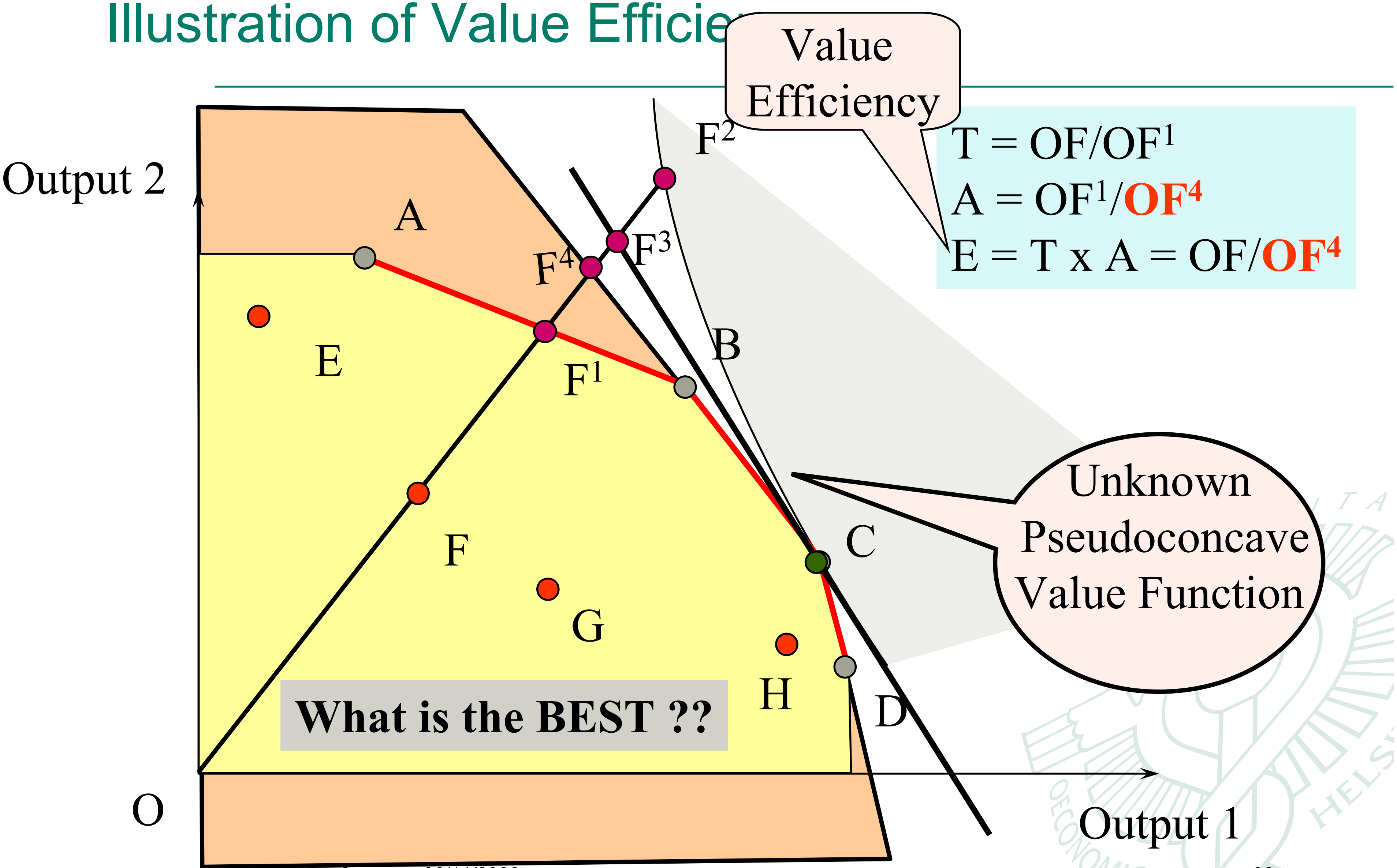


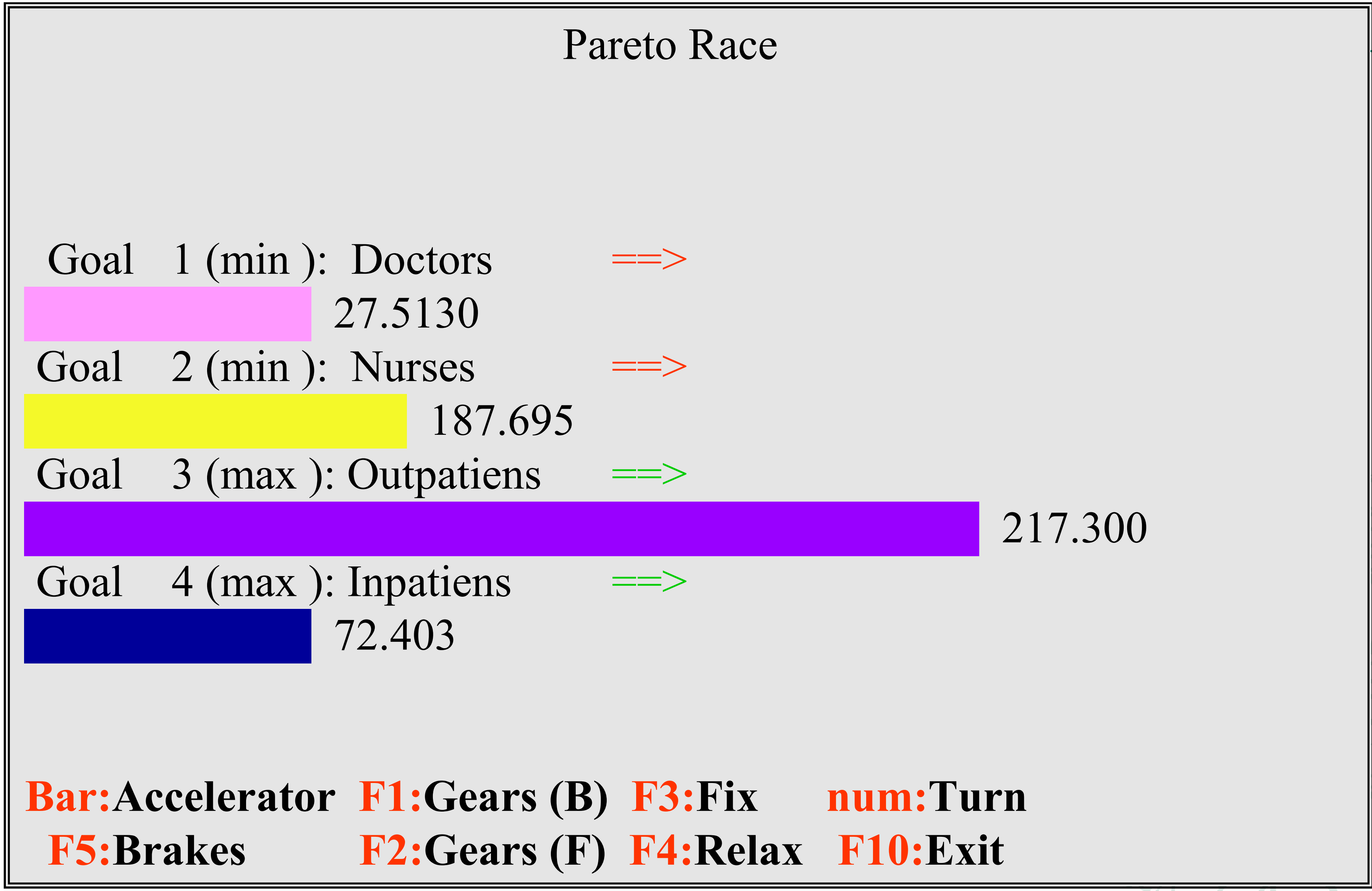
Illustration of Value Efficiency



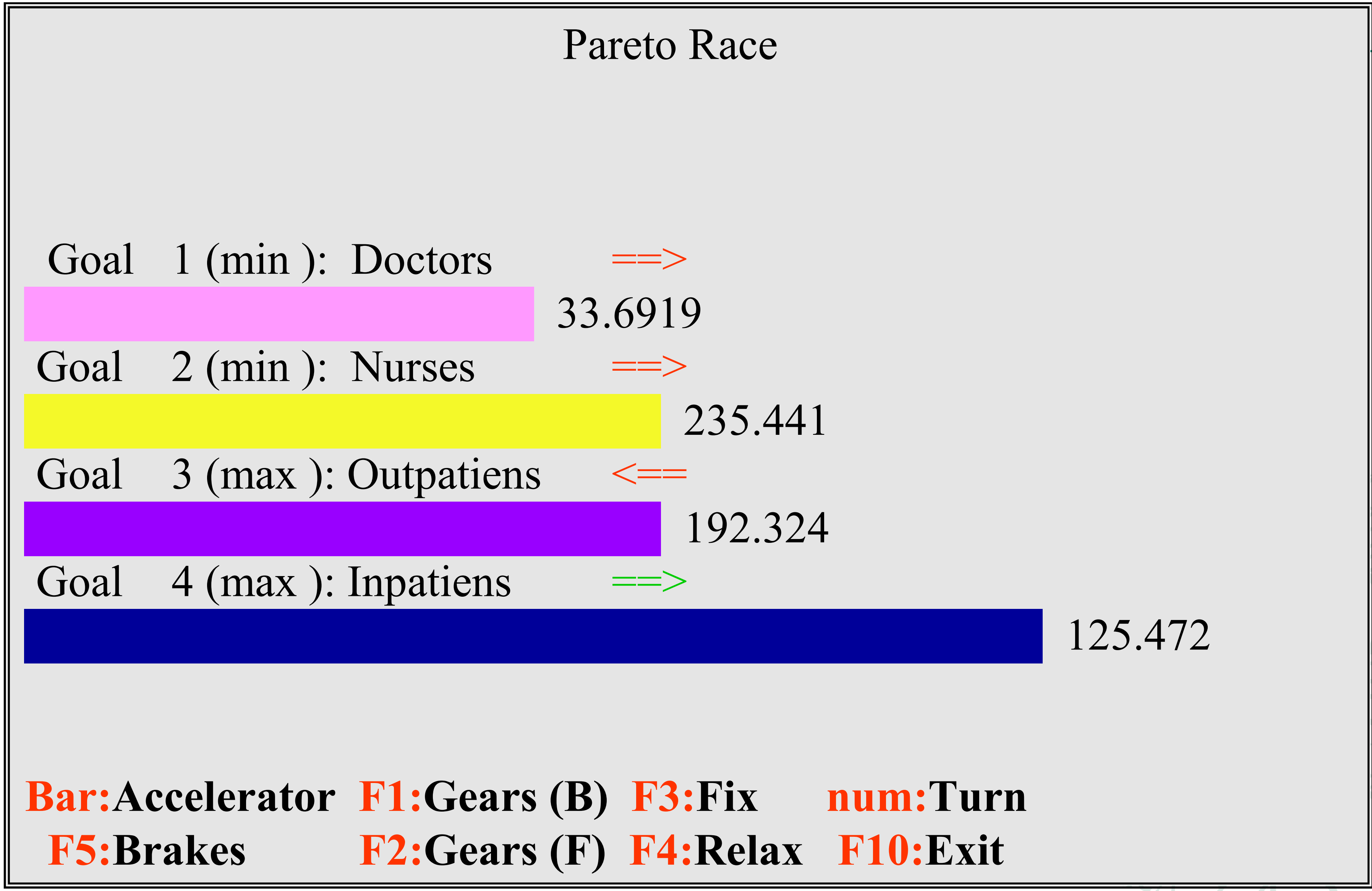
Hospital Case (Cooper et al. 2000)

Hospitals	A	B	C	D	E	F	G	H	I	J	K	L
Doctors	20	19	25	27	22	55	33	31	30	50	53	38
Nurses	151	131	160	168	158	255	235	206	244	268	306	284
Outpatiens	100	150	160	180	94	230	220	152	190	250	260	250
Inpatients	90	50	55	72	66	90	88	80	100	100	147	120

Pareto Race

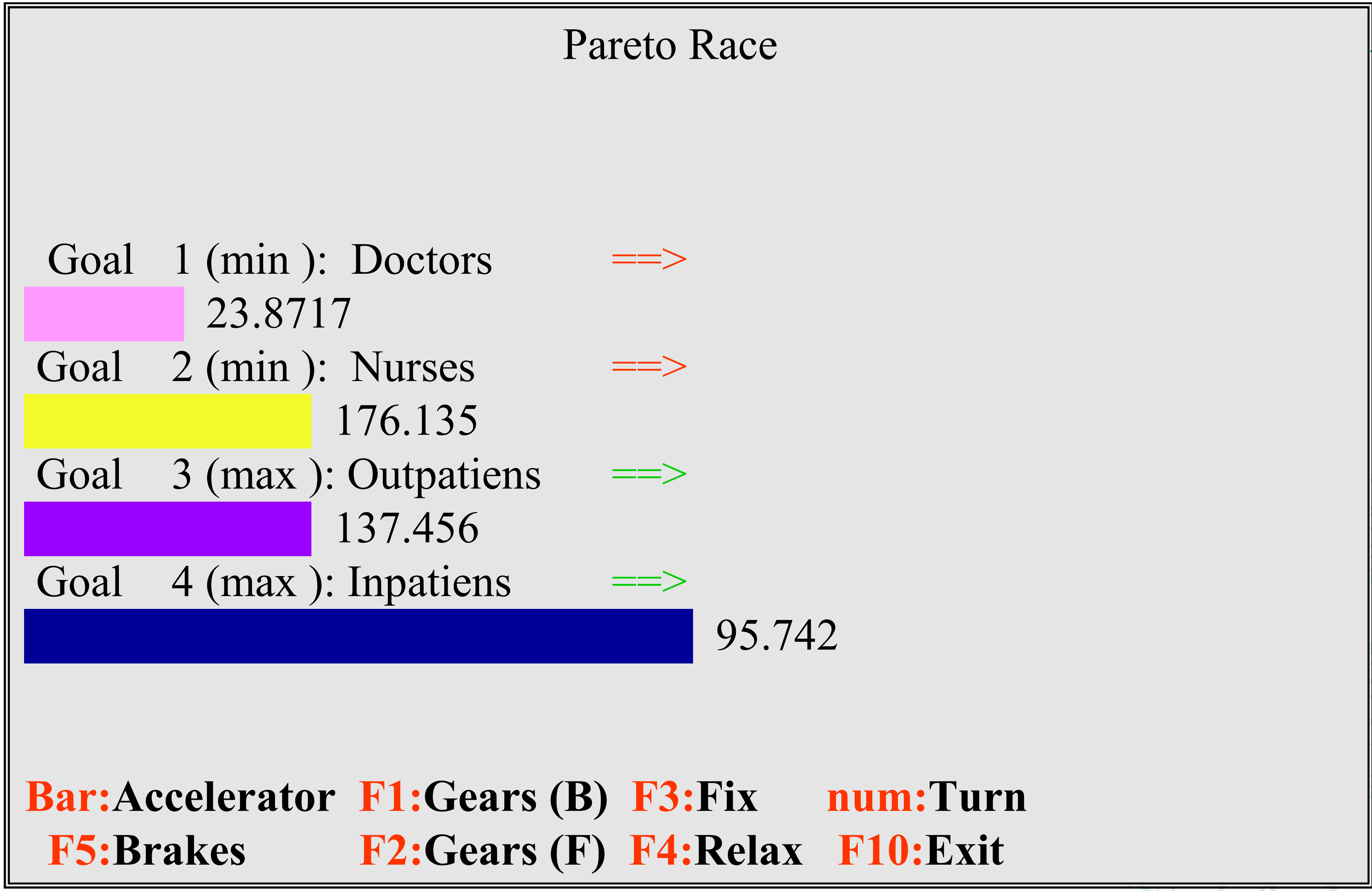

Pareto Race
Goal 1 (min): Doctors ==>
27.5130
Goal 2 (min): Nurses ==>
187.695
Goal 3 (max): Outpatiens ==>
217.300
Goal 4 (max): Inpatiens ==>
72.403
Bar:Accelerator F1:Gears (B) F3:Fix num:Turn
F5:Brakes F2:Gears (F) F4:Relax F10:Exit

Pareto Race



Pareto Race
Goal 1 (min): Doctors ==>
33.6919
Goal 2 (min): Nurses ==>
235.441
Goal 3 (max): Outpatiens <==
192.324
Goal 4 (max): Inpatiens ==>
125.472
Bar:Accelerator F1:Gears (B) F3:Fix num:Turn
F5:Brakes F2:Gears (F) F4:Relax F10:Exit

Pareto Race


Pareto Race
Goal 1 (min): Doctors ==>
23.8717
Goal 2 (min): Nurses ==>
176.135
Goal 3 (max): Outpatiens ==>
137.456
Goal 4 (max): Inpatiens ==>
95.742
Bar:Accelerator F1:Gears (B) F3:Fix num:Turn
F5:Brakes F2:Gears (F) F4:Relax F10:Exit

Hyper Markets (25)

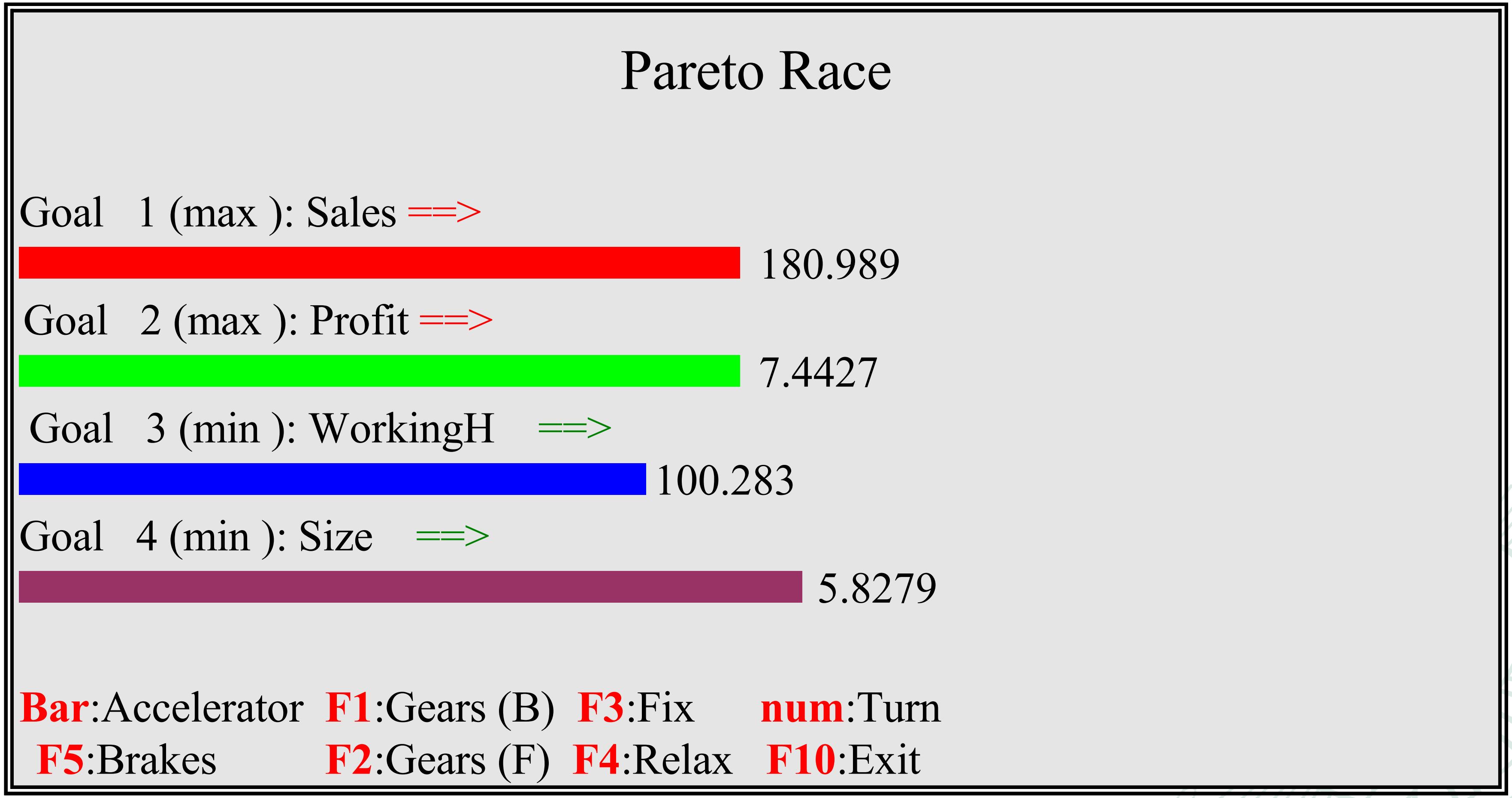
	Outputs		Inputs	
	Sales	NetProfit	Man Hour	SalesSpace
FIRM1	115.266	1.708	79.056	4.986
FIRM2	75.191	1.811	60.096	3.3
FIRM3	225.454	10.393	126.699	8.117
FIRM4	185.581	10.417	153.857	6.695
FIRM5	84.52	2.357	65.684	4.735
FIRM6	103.328	4.347	76.83	4.083
FIRM7	78.755	0.162	50.157	2.531
FIRM8	59.327	1.299	44.771	2.47
FIRM9	65.718	1.485	48.058	2.324
FIRM10	163.178	6.261	89.702	4.911
FIRM11	70.679	2.802	56.923	2.24
FIRM12	142.648	2.745	112.637	5.42
FIRM13	127.767	2.701	106.869	6.281
FIRM14	62.383	1.418	54.932	3.135
FIRM15	55.225	1.375	48.809	4.43
FIRM16	95.925	0.742	59.188	3.979
FIRM17	121.604	3.059	74.514	5.318
FIRM18	107.019	2.983	94.596	3.691
FIRM19	65.402	0.618	47.042	3.001
FIRM20	70.982	0.005	54.645	3.865
FIRM21	81.175	5.121	90.116	3.31
FIRM22	128.303	3.887	95.241	4.245
FIRM23	134.989	4.728	80.079	3.786
FIRM24	98.931	1.861	68.703	2.985
FIRM25	66.743	7.409	62.282	3.1

A Model for Searching the Most Preferred Solution

	FIRM1	FIRM2	…	FIRM23	FIRM24	FIRM25		
Sales	115.266	75.191		134.989	98.931	66.743	$\rightarrow$	max
NetProfit	1.708	1.811	…	4.728	1.861	7.409	$\rightarrow$	max
ManHour	79.056	60.096		80.079	68.703	62.282	$\rightarrow$	min
SalesSpace	4.986	3.3		3.786	2.985	3.1	$\rightarrow$	min
λ-constr.	1	1	…	1	1	1	=	1

HSE

Moving on the Efficient Frontier



	Efficiency	Value Efficiency Score				
		Firm # 3	Firm # 8	Firm # 10	Firms # 3 & 10	Firms # 3 & 8 ⇒ # 8 & 10 & 25
FIRM1	0.821	0.794	0.768	0.821	0.794	0.720
FIRM2	0.772	0.663	0.772	0.772	0.663	0.758
FIRM3	1.000	1.000	0.931	1.000	1.000	0.931
FIRM4	1.000	1.000	0.661	1.000	1.000	0.661
FIRM5	0.769	0.689	0.769	0.769	0.689	0.757
FIRM6	0.806	0.780	0.805	0.806	0.780	0.805
FIRM7	1.000	0.815	1.000	1.000	0.815	0.953

	Efficiency	Value Efficiency Score				
RM5	0.769	0.689	0.769	0.769	0.689	0.757
RM6	0.806	0.780	0.805	0.806	0.780	0.805
RM7	1.000	0.815	1.000	1.000	0.815	0.953
RM8	1.000	0.731	1.000	1.000	0.731	1.000
RM9	1.000	0.743	1.000	1.000	0.743	1.000
RM10	1.000	1.000	1.000	1.000	1.000	1.000
RM11	1.000	0.749	0.913	0.966	0.749	0.913
RM12	0.824	0.824	0.613	0.824	0.824	0.592
RM13	0.673	0.673	0.588	0.673	0.673	0.558
RM14	0.736	0.596	0.736	0.736	0.596	0.722
RM15	0.803	0.623	0.803	0.803	0.623	0.802
FIRM16	0.978	0.858	0.948	0.978	0.858	0.881
FIRM17	0.930	0.884	0.914	0.930	0.884	0.887
FIRM18	0.817	0.767	0.613	0.807	0.767	0.605
FIRM19	0.969	0.716	0.969	0.959	0.716	0.915
FIRM20	0.804	0.681	0.774	0.804	0.681	0.703
FIRM21	0.858	0.793	0.600	0.846	0.793	0.600
FIRM22	0.876	0.854	0.724	0.876	0.854	0.716
FIRM23	1.000	0.960	0.973	1.000	0.960	0.973
FIRM24	0.973	0.787	0.838	0.907	0.787	0.819
FIRM25	1.000	1.000	1.000	1.000	1.000	1.000

Kirjallisuutta

1. COELLI, T., RAO, D.S.P. & BATTESE, G. (1998), An Introduction to Efficiency and Productivity Analysis, Kluwer
2. COOPER, W.W., SEIFORD, L., and TONE, K. (2000): Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software", Kluwer Academic Publishers.
3. JORO, T., KORHONEN, P. AND WALLENIUS, J. (1998): "Structural Comparison of Data Envelopment Analysis and Multiple Objective Linear Programming", **Management Science**, Vol. 44, N:o 7, pp. 962-970.
4. HALME, M., JORO, T., KORHONEN, P., SALO, S., AND WALLENIUS, J. (1999): "A Value Efficiency Approach to Incorporating Preference Information in Data Envelopment Analysis", **Management Science**, Vol. 45, N:o 1, pp. 103-115.

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Artist:

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