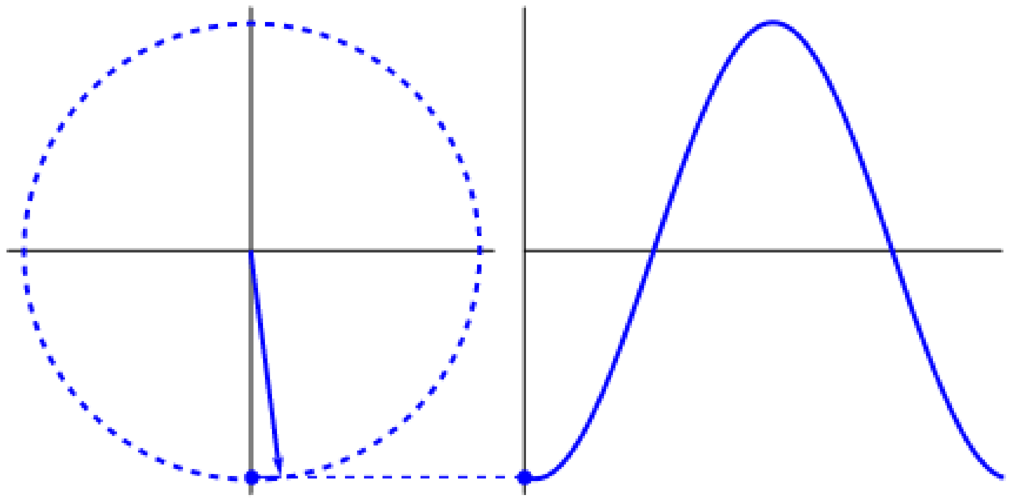


Complex Impedance in a Resistor-Capacitor Circuit

1129 Sergey Ichtchenko

Complex Numbers

Cartesian form

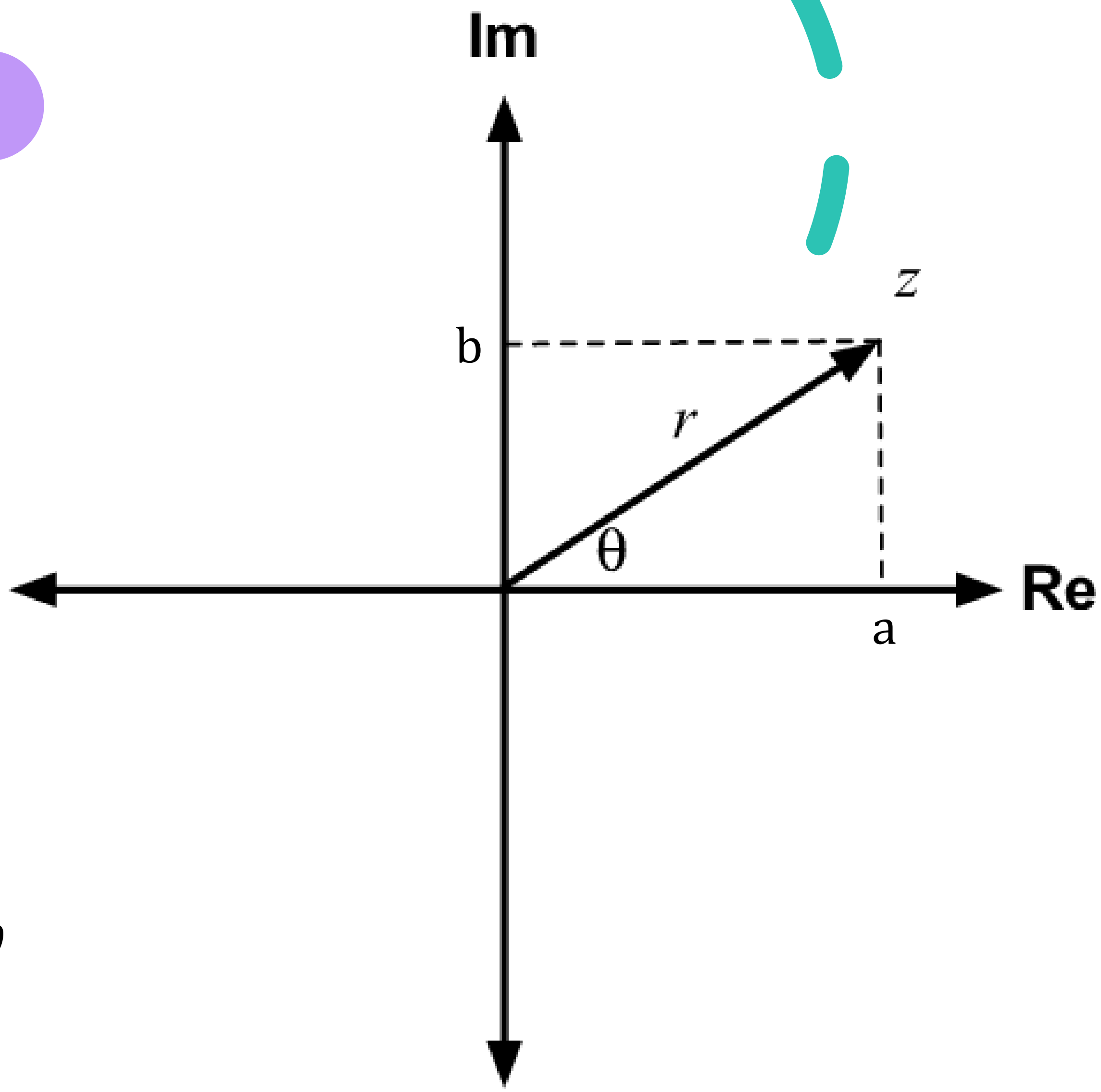
- Have a real part a
- Have an imaginary part b

$$z = a + bi$$

Polar/exponential form

- Have a modulus θ
- Have an argument r

$$z = r(\cos(\theta) + i\sin(\theta)) = rcis(\theta) = re^{i\theta}$$



Complex Impedance

Cartesian form

- Has a real part $Z_R = R$
- Has an imaginary part $Z_C = -\frac{1}{2\pi f C}$

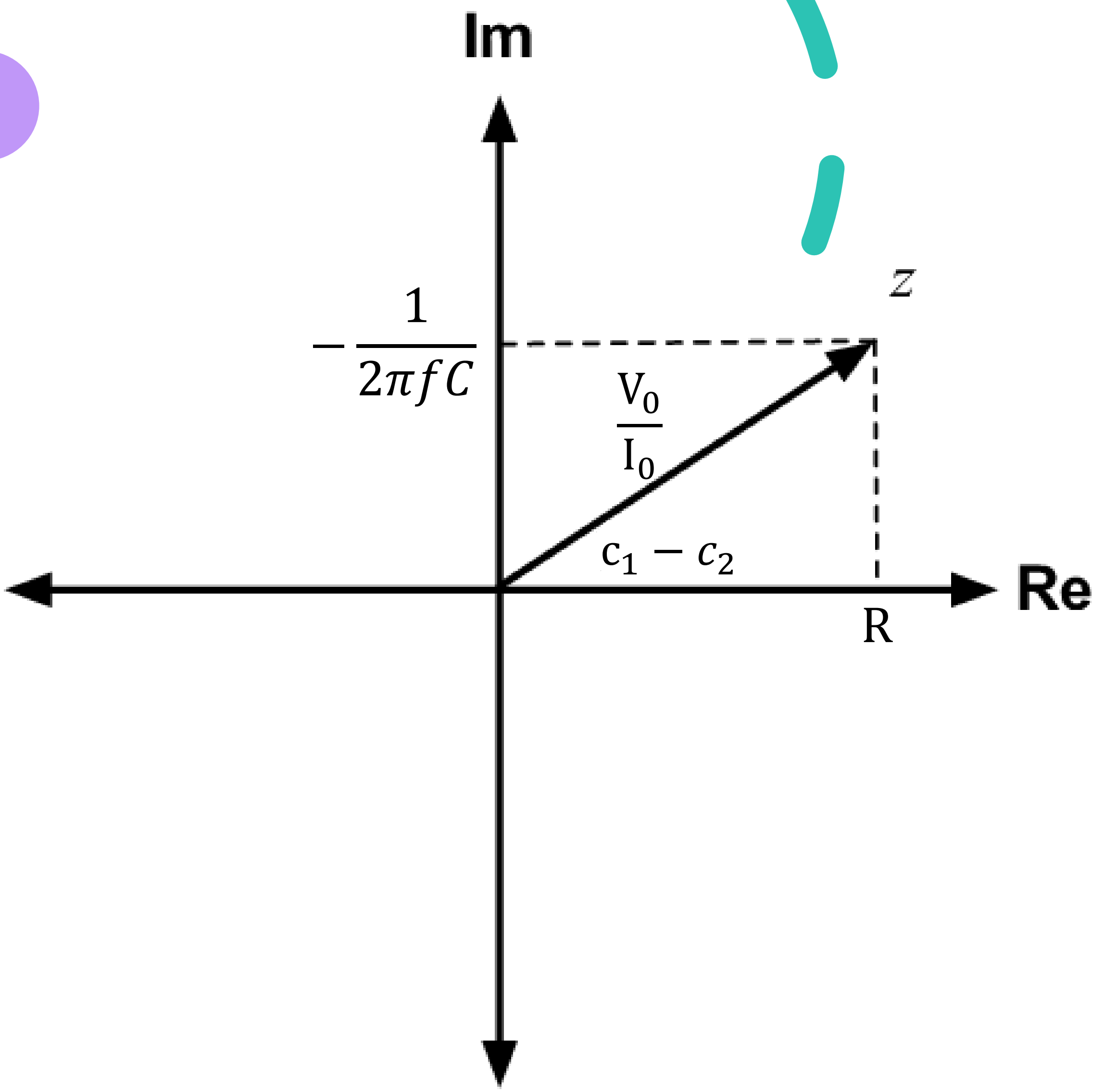
$$z = R + -\frac{1}{2\pi f C} i$$

Polar/exponential form

- Has a modulus θ which corresponds to the phase shift between voltage and current
- Has an argument r which corresponds to the quotient of amplitudes of voltage and current

$$\hat{V} = V_0 e^{i(\omega t + c_1)}$$
$$\hat{I} = I_0 e^{i(\omega t + c_2)}$$

$$z = \frac{V_0}{I_0}(\cos(c_1 - c_2) + i \sin(c_1 - c_2)) = \frac{V_0}{I_0} cis(c_1 - c_2) = \frac{V_0}{I_0} e^{i(c_1 - c_2)}$$



Aim

To verify whether the two ways of measuring impedance are equivalent

Research question

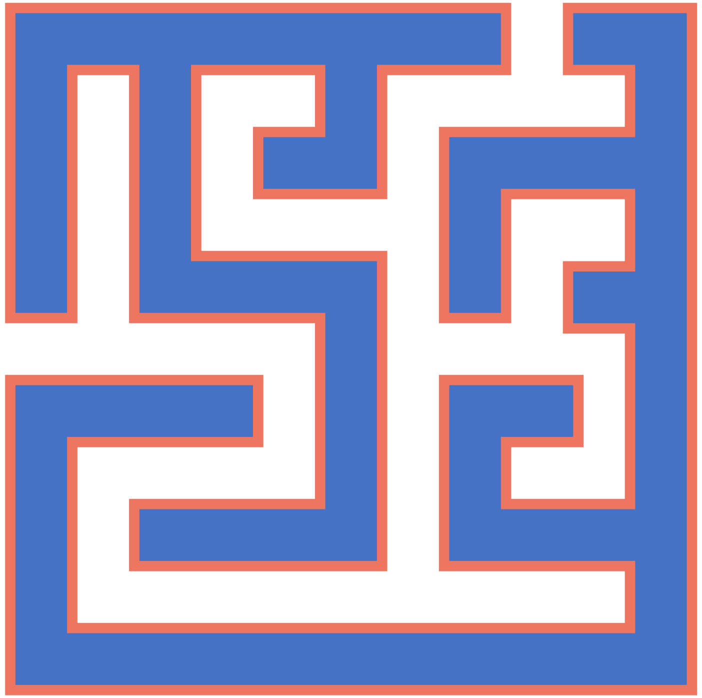
How does the capacitance of an RC-circuit with AC affect its complex impedance?

Hypothesis

The two forms of measuring impedance would be equal. Therefore, the capacitance would affect the impedance by the equations

$$\theta = \tan^{-1}\left(\frac{1}{-2\pi f C R}\right) = c_1 - c_2$$

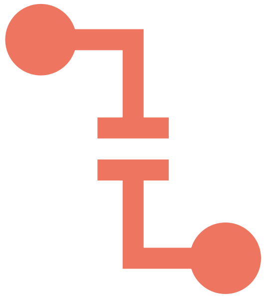
$$r = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2 R^2}} = \frac{V_0}{I_0}$$



Personal relevance

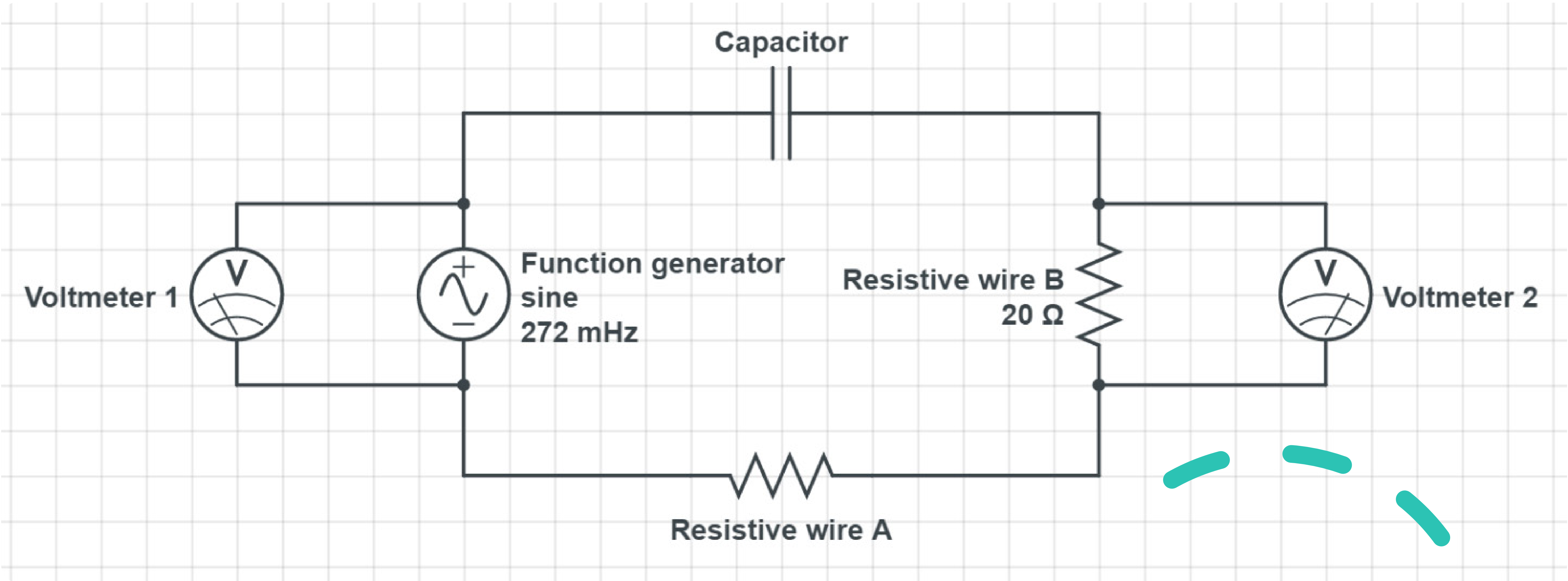
- How can something based in reality use complex numbers?
- Applying familiar maths to a real-life setting
- Expanding my understanding of AC

Experimental design



Independent variable: The capacitance of the circuit

Dependent variable: The complex impedance of the circuit

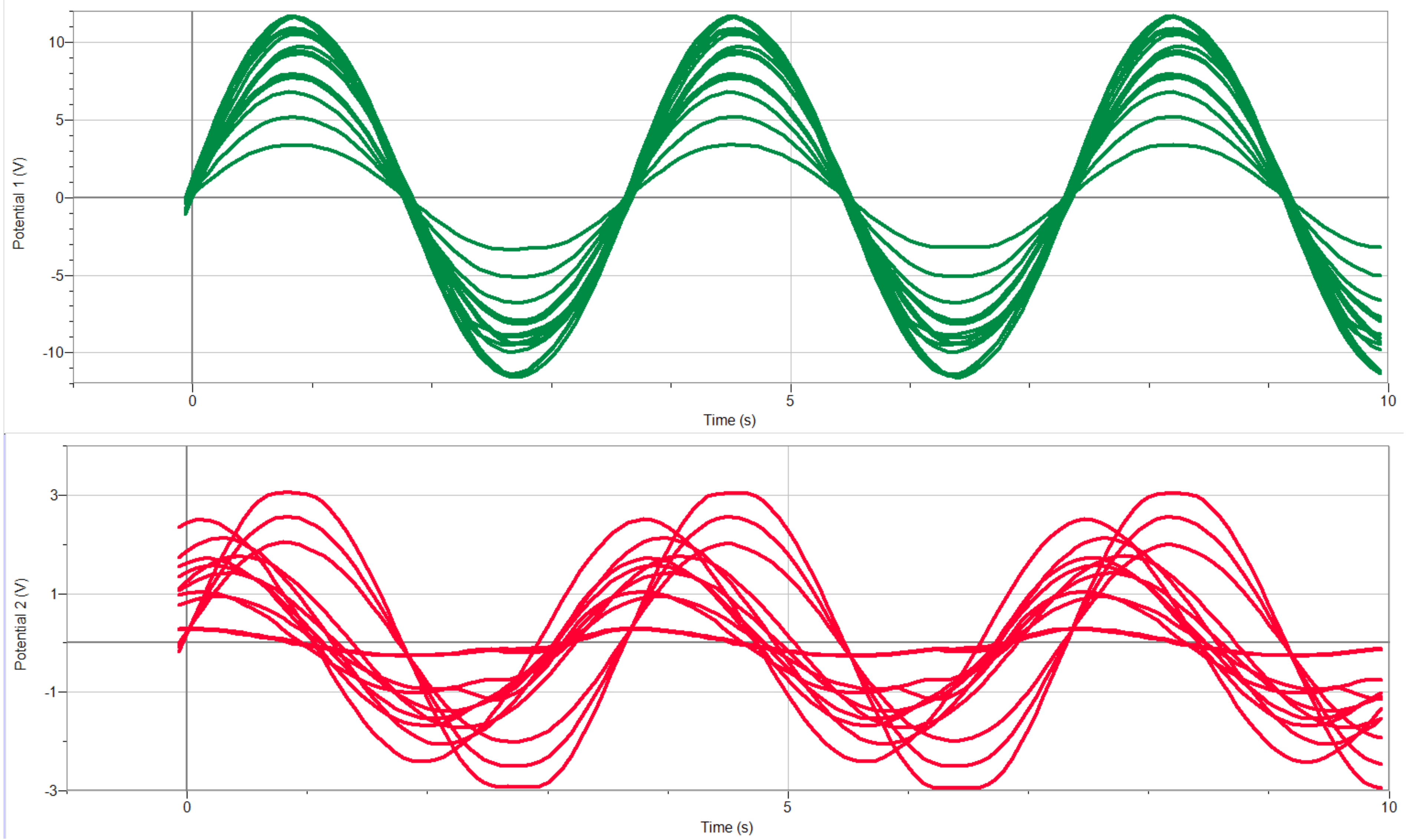


Circuit diagram

Method

1. Set up the components as shown in the circuit diagram
2. With the function generator off, calibrate and zero the voltmeters
3. Turn the frequency of the function generator to 272mHz and the amplitude to maximum (12V). The function generator may output a lower voltage than 12V depending on the impedance of the circuit.
4. Set the measurement to start when the voltage from Voltmeter 1 is increasing across 0V
5. Measure for 10s to get a suitable number of oscillations
6. Change the capacitor and repeat the measurement for each of the five capacitances
7. Change resistive wire A and repeat all the measurements for each of the three resistances

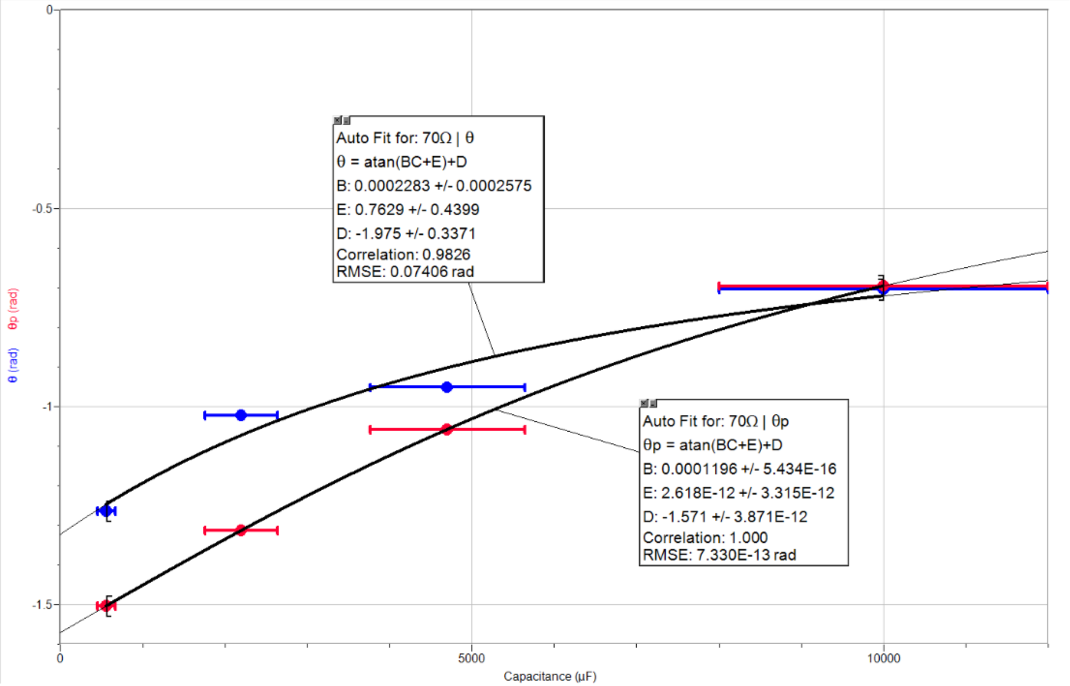
Raw data

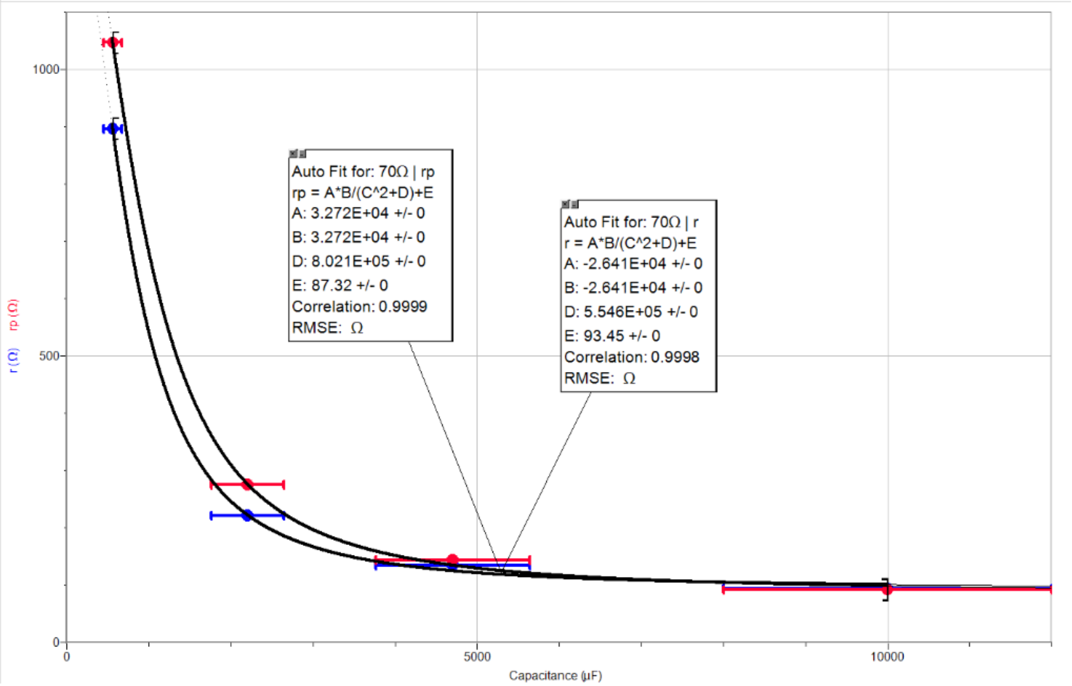

Potential 1 (V)
10
5
0
-5
-10
0
5
10
Time (s)
Potential 2 (V)
3
1
-1
-3
0
5
10
Time (s)

Data processing

- Sinusoidal curve fits for all the raw data
- Calculations
  - Predicted value of impedance from resistance and capacitance
  - Experimental value of impedance from phase shift and amplitude quotient
- Comparisons
  - An arctangential line of best fit through arguments
  - A square-of-inverse-square line of best fit through moduli

Processed data





Resistance (Ω)	Capacitance (µF)	A_1 (V)	A_2 (V)	C_1 (rad)	C_2 (rad)	r (Ω)	θ (rad)	rp (Ω)	θp (rad)	Δθ (rad)	Δr/r (Ω/Ω)
20	0	3.478	0.1577	0.08197	0.074	22.055	0.008	20.000	0.000	0.012	0.006
20	560	11.52	0.01224	0.1007	1.297	941.176	-1.196	1045.080	-1.552	0.060	0.028
20	2200	10.24	0.05318	0.08865	1.206	192.554	-1.117	266.722	-1.496	0.053	0.025
20	4700	9.587	0.08457	0.006597	1.233	113.362	-1.226	126.094	-1.412	0.019	0.009
20	10000	8.021	0.1266	0.1106	1.333	63.357	-1.222	61.837	-1.241	0.006	0.003
40	0	5.262	0.1294	0.9916	0.09908	40.665	0.893	40.000	0.000	0.007	0.004
40	560	11.55	0.0127	0.1276	1.394	909.449	-1.266	1045.654	-1.533	0.043	0.020
40	2200	10.33	0.05318	0.09776	1.206	194.246	-1.108	268.963	-1.422	0.038	0.018
40	4700	9.461	0.07744	0.04194	1.158	122.172	-1.116	130.765	-1.260	0.011	0.012
40	10000	7.839	0.1046	0.0962	1.06	74.943	-0.964	70.879	-0.971	0.006	0.003
70	0	6.835	0.101	0.1268	0.1301	67.673	-0.003	70.000	0.000	0.007	0.003
70	560	11.54	0.01287	0.04138	1.306	896.659	-1.265	1047.231	-1.504	0.039	0.018
70	2200	10.35	0.04694	0.04299	1.066	220.494	-1.023	275.029	-1.313	0.026	0.012
70	4700	9.38	0.06992	0.02006	0.9722	134.153	-0.952	142.827	-1.059	0.007	0.004
70	10000	8.091	0.08733	0.08873	0.794	92.649	-0.705	91.235	-0.696	0.006	0.003

Conclusion

Capacitance in an RC circuit makes the argument of complex impedance negative, observable as a negative phase shift between voltage and current, or the current lagging behind the voltage. A higher capacitance makes the magnitude of the argument smaller. A higher capacitance also makes the modulus of the complex impedance smaller, observable as a smaller ratio of amplitudes of voltage to current.

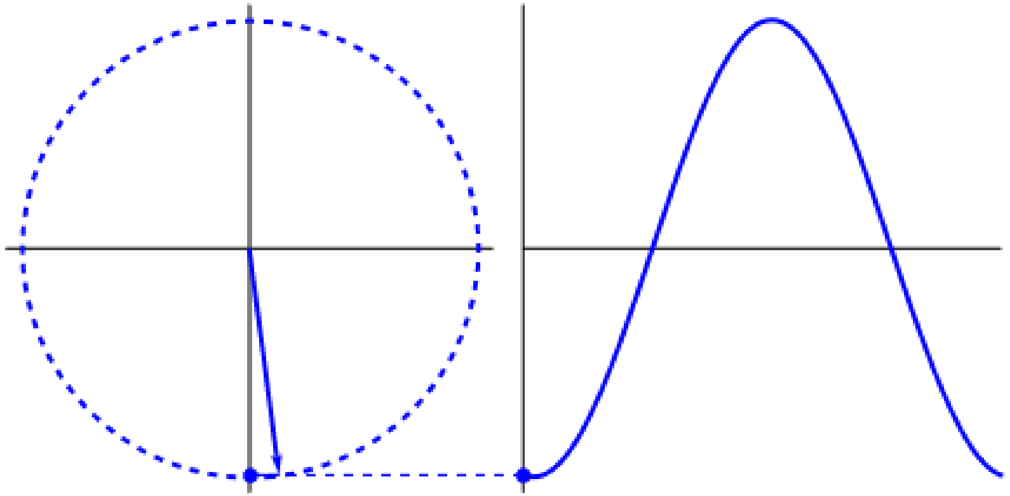
Conclusion (in English)

The two ways of measuring impedance are identical

Considerations

- High, 20% tolerance on capacitors
- Imprecise voltage measurements over a small resistive load due to the lack of a sensitive enough ammeter
- Function generator producing variable amplitudes

However, even with these considerations, I could deduce reliable results with a strong correlation between variables



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