ONE-SIDED MATCHING – APPLICATION TO KIDNEY EXCHANGE

Hannu Vartiainen
University of Helsinki

HOW DO MARKETS WORK?

- Commodity markets are simple: buyers choose what they want from what they can afford
- Price mechanism
  - coordination
  - efficiency
  - transparency and safety
- Requires
  - property rights that can be transferred
  - prices and “money”
- But many “matching” markets, e.g. school choice, marriage, organ transplants, cannot employ money (why?)

=> **price mechanism cannot be applied**

MARKETPLACES

- In the absence of price mechanism, matching markets need alternative coordination device – a clearinghouse
  - where both sides of the markets meet
  - who gets what is determined
  - is controllable by the authorities
- To be efficient, clearinghouse should create a market that is
  - thick but avoids congestion
  - safe and avoids “gaming”
- **How to design one?**

TYPES OF MATCHING PROBLEMS

- Two sided-matching: both sides of the market make choices
  - job market
  - school choice
  - marriage
- One-sided matching: only one side of the market makes choices
  - housing
  - organ transplant

ONE-SIDED MATCHING - HOUSING MARKETS

- Shapley and Scarf (1974): canonical one-sided matching model, a “housing market”
- Set $N = \{1,\ldots,n\}$ of agents, each endowed with an indivisible good, a “house”
- Agent i has preferences P_i over all n houses, $\boldsymbol{P}$ being the set of all preferences
- No money, trade is feasible only in houses
- **A matching** function $\mu: N \rightarrow N$ assigns each agent a house, $\boldsymbol{M}$ being the set of all matchings
- **A mechanism** is a procedure $\mu(\cdot:\cdot): \boldsymbol{P} \rightarrow \boldsymbol{M}$ that selects a matching for each preference profile, $\mu(i:P)$ being the match of agent i under preference profile P in $\boldsymbol{P}$

DESIRABLE PROPERTIES

- Matching $\mu(i{:}P)$ of i now depends on all agents' preferences – or preferences that they reveal to the mechanism
- *How to guarantee that mechanism* $\mu(\cdot{:}\cdot)$ *works as intended*?
- Matching μ is **individually rational** if every agent obtains a house at least as good as his own.
- Matching μ is in **the core** if no coalition of agents B can block the matching – trade among themselves while making every agent in B better off (all weakly, some strictly)

Fact: *Any core matching is individually rational and Pareto optimal*

- Matching μ is **strategy-proof** if it is every agent's dominant strategy to report his true preferences to μ, given the outcome it induces

$$\mu(i{:}P)\ P_i\ \mu(i{:}P_i, P_{-i}) \quad \text{for all } (P_i, P_{-i}), \text{ for all } i$$

SOLUTION - TTC

Top Trading Cycle - TTC (Gale):

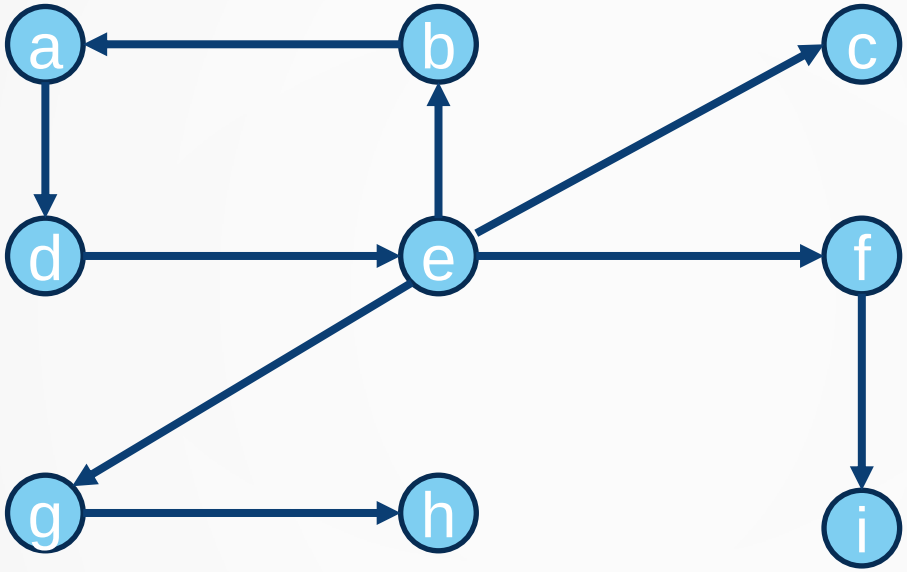
Step 1: Each agent points to her most preferred house (and each house points to its owner). There is at least one cycle in the resulting directed graph (a cycle may consist of an agent pointing to her own house.) In each such cycle, the corresponding trades are carried out and these agents are removed from the market together with their assignments.

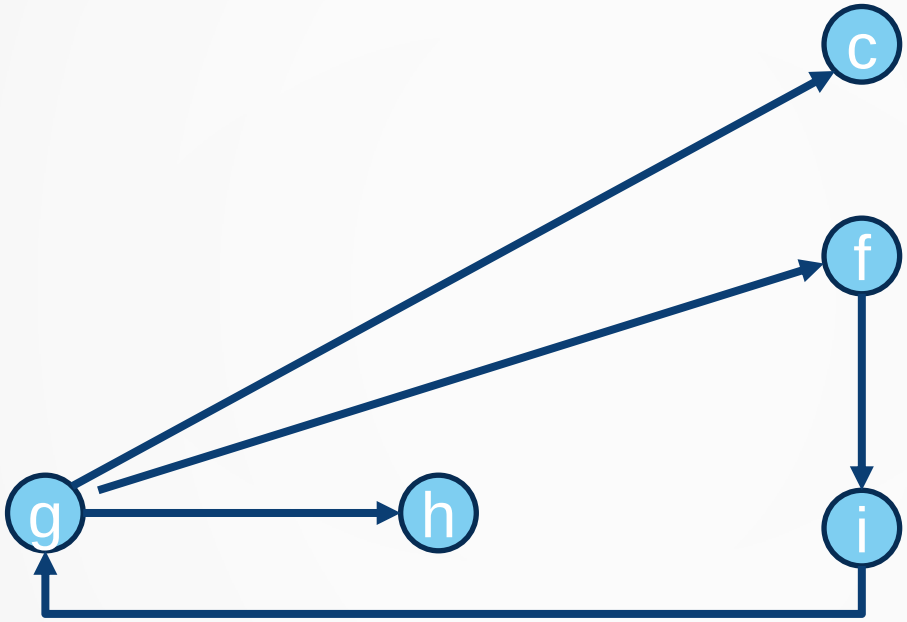
…

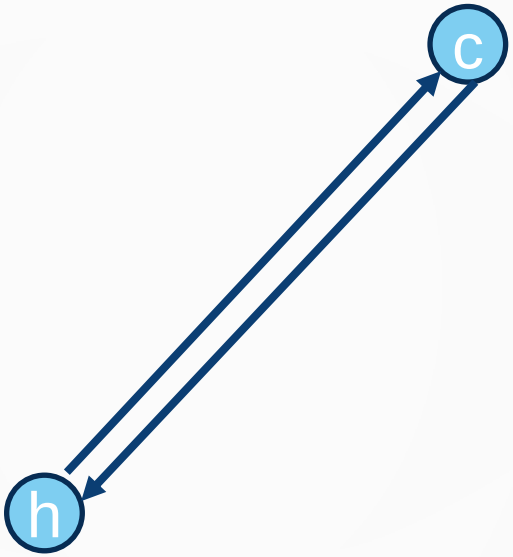
Step *t*: each agent points to her most preferred house that remains on the market…

...

$i \rightarrow j$ = "j prefers i's house"




c
f
g
h
i


c
h

RESULTS

Theorem [Shapley and Scarf 1974]: *TTC is in the core for any preference profile*

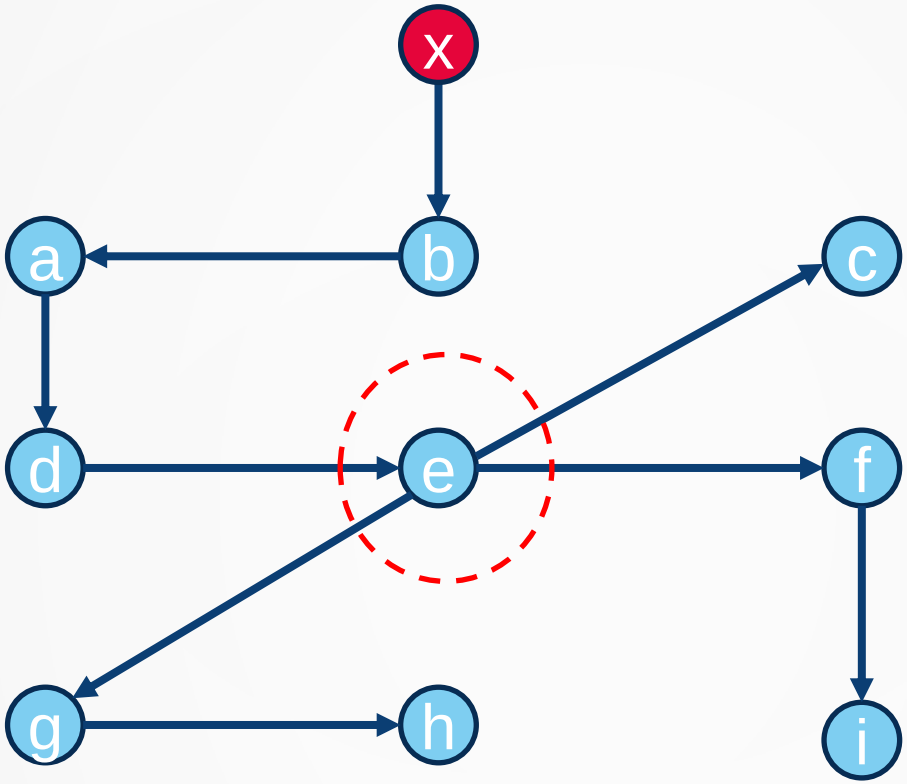
Theorem [Roth and Postlewaite 1977] *TTC is the unique matching in the core (under strict preferences)*

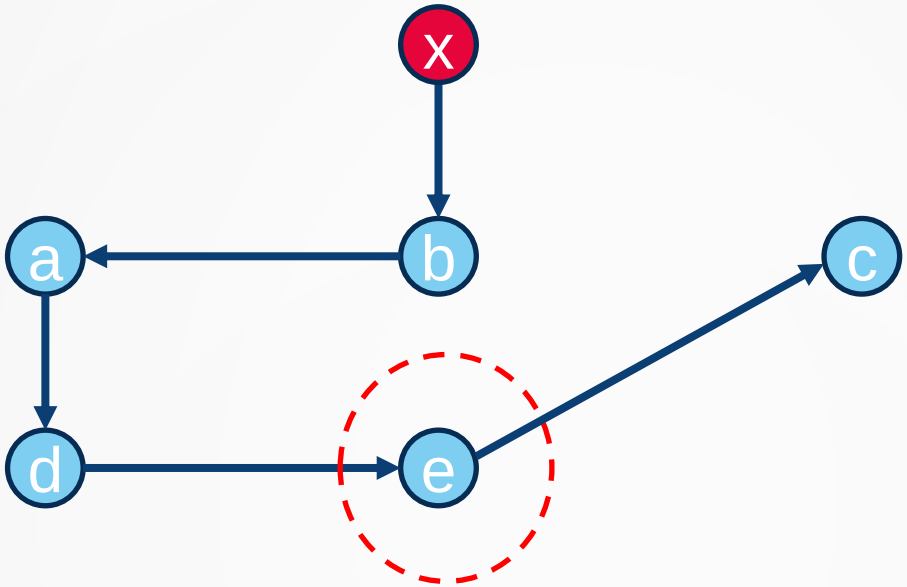
Theorem [Roth 1972]: *TTC is strategy-proof*

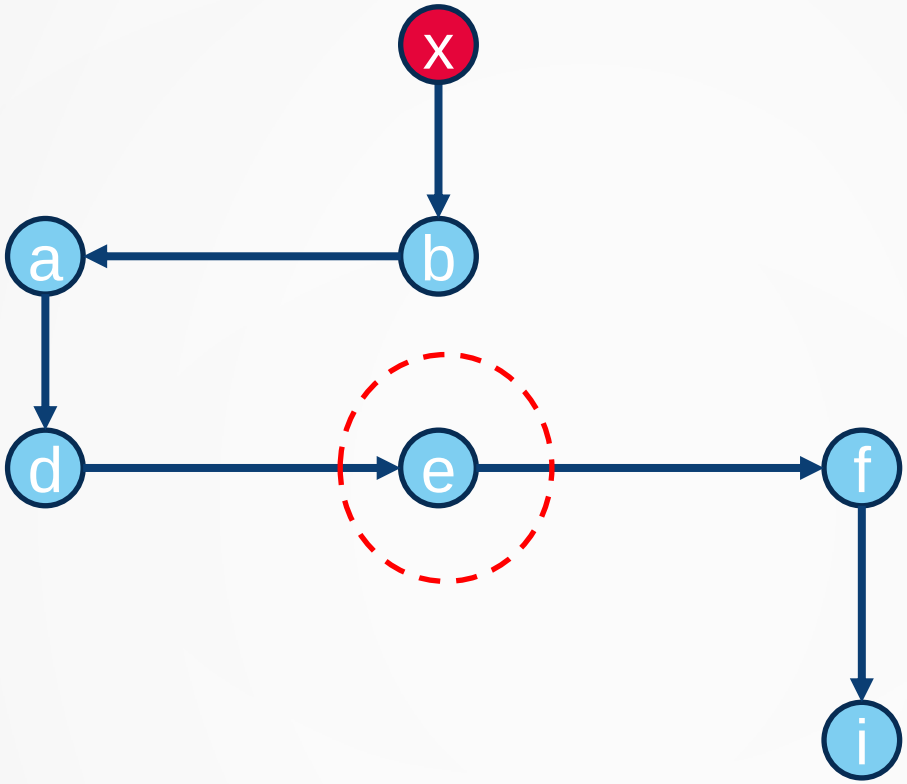
- TTC or its variants extensively used in practical applications, e.g. public housing allocation, office allocation, school choice,…

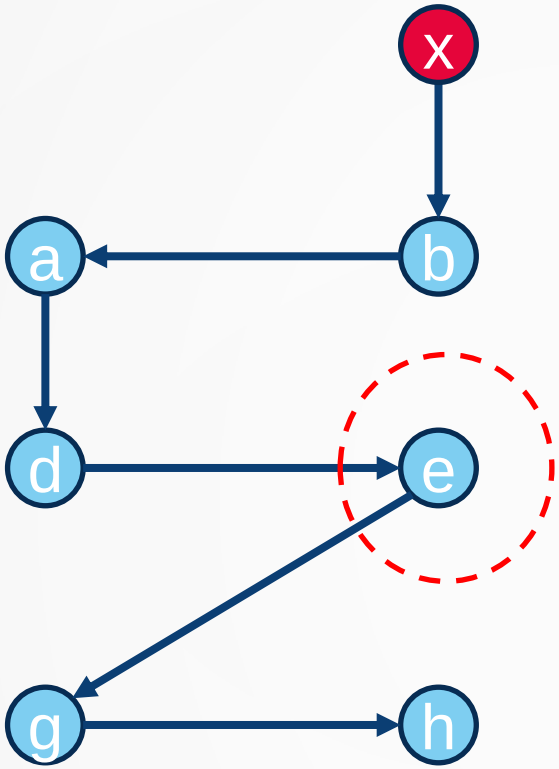
EXTENSIONS

- TTC is based on cycles – participating one is individually optimal at each stage => strategy-proofness
- If new houses come to market – without a prior owner – cycles need not form: new houses are not directed and hence multiple chains may form
- Can the good properties of the TTC be saved with chains?


x
a
b
c
d
e
f
g
h
i


x
a
b
c
d
e


x
a
b
d
e
f
i


x
a
b
d
e
g
h

Theorem [Roth et al 2004]: *Strategy-proof and Pareto optimal "TTCC" mechanisms exist for selecting cycles and chains*

- That is, it is possible to integrate cycles and chains in a way that makes it safe for house owners to reveal information on their preferences

KIDNEY EXCHANGE

- Two treatments for patients with end-stage kidney disease: dialysis and transplantation
- Dialysis requires a strong dependence and has difficult side effects (physical and psychological), hence transplantation by far the best treatment
  - direct costs of dialysis 50 000 €/year, indirect costs much higher as the patient not able to work
  - one-time cost of a transplantation 25 000 € after which the recipient is also able to return to normal life
  - transplantation patients 10 years longer life expectancy
- In Finland, appr. 500 new patients reach the ESKD but the number of annual transplantations is 150-200

- The median waiting time for a kidney (of those who eventually receive one) 1,5 years
  - during the waiting time, dialysis treatment
  - annually 5-10 percent of waitlisted die without receiving a kidney.
- Almost all the transplanted kidneys in Finland are received from cadaveric donors
- Due to demographics and increasing prevalence of kidney related diseases, the demand for kidneys steadily increases
- However, by a survey by the Finnish Kidney and Liver Association, one half of the 350 waitlisted Finnish ESKD patients **would have a donor**
- **The problem**: immunological incompatibility of patients and the intended donors

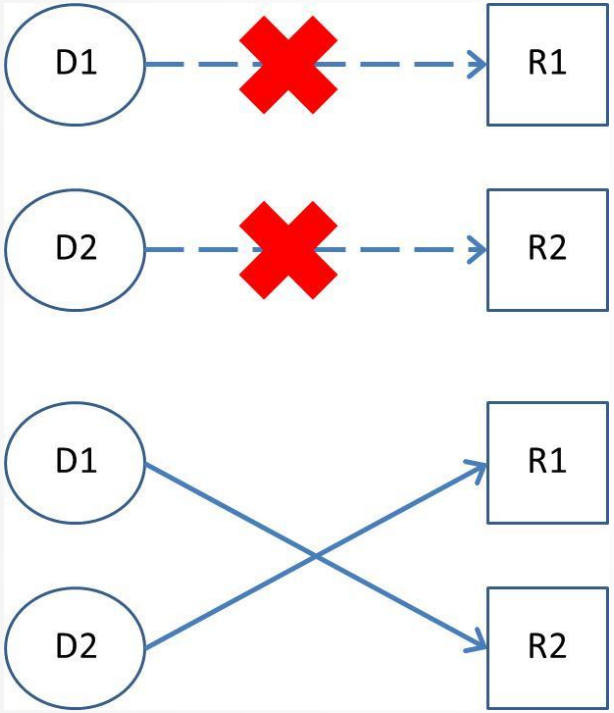
Kidney incompatibility - medical background

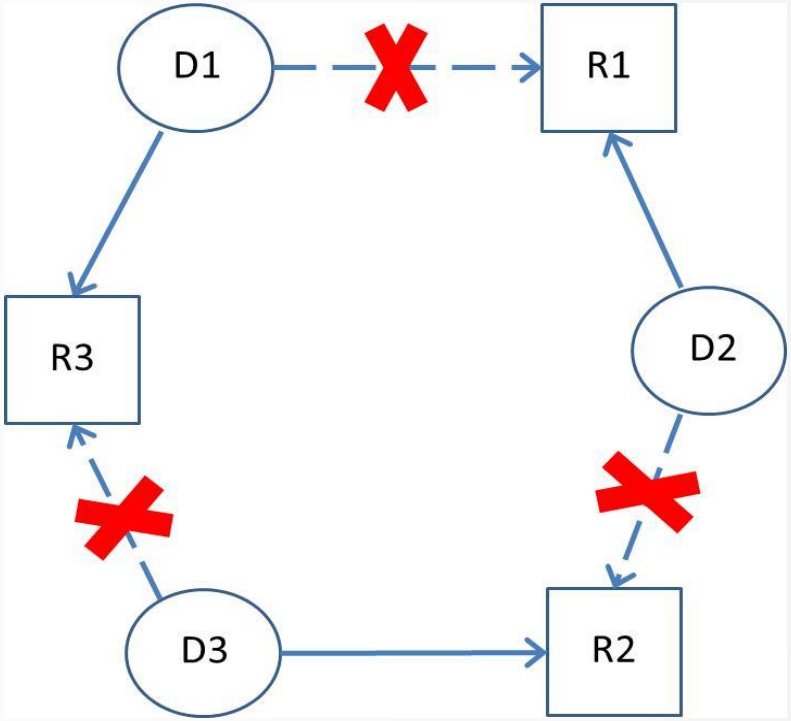
Blood-type incompatibility: Blood type O kidneys can be donated to any patient, but type O patients can only receive type O kidneys. Type A and type B kidneys can donate to patients of their own blood type or type AB patients. Type AB kidneys can only be donated to type AB patients.

Tissue-type incompatibility: The tissue compatibility is determined by the human leukocyte antigen (HLA) sensitization of the patient and the donor. HLA type consists of a combination of six proteins. The more of a mismatch between the patient and donor's HLA types, the less likely a transplant will be successful. Furthermore, it is possible that a patient's body contains antibodies targeted against the donor's HLA type. This "positive cross match", which is fairly typical with mothers and their children, precludes transplantation (in general, however, genetic proximity increases the likelihood of blood type and tissue type compatibility.)

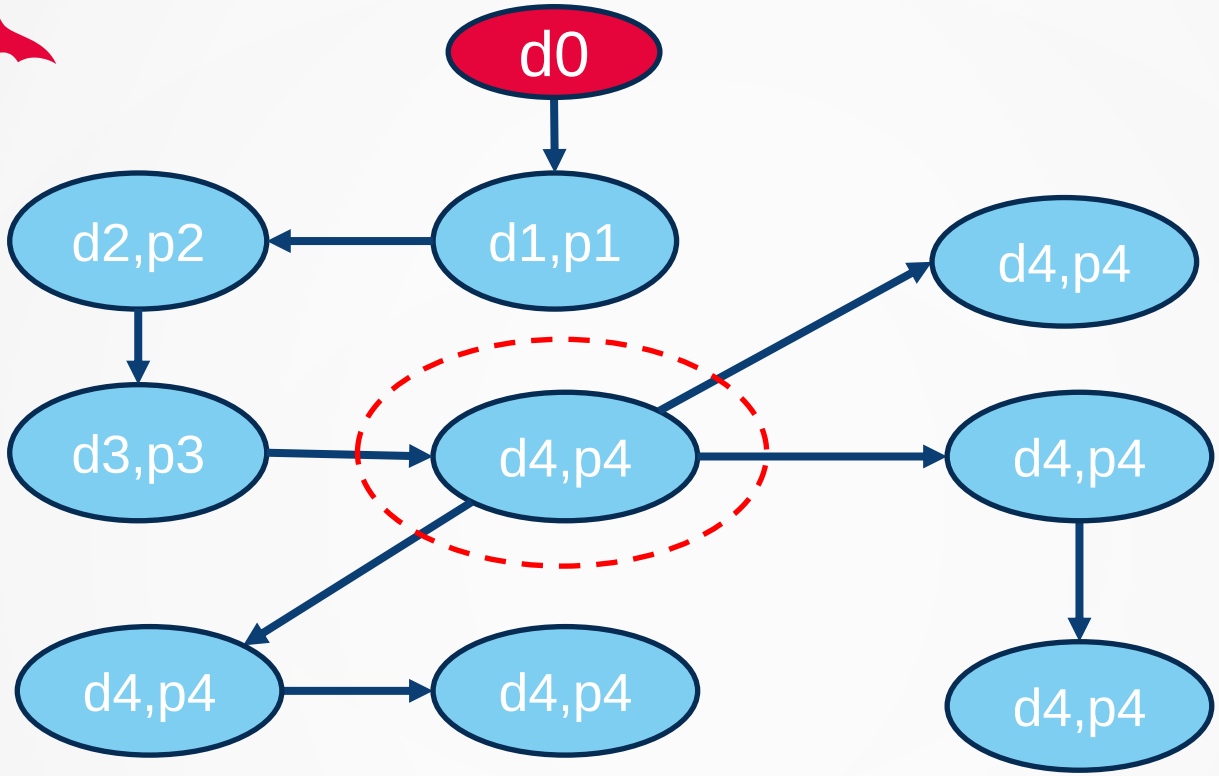
KIDNEY EXCHANGE MECHANISM

- ***Kidney matching mechanism*** provides a partial remedy to the incompatibility problem
- The working idea: to enhance exchange of kidneys between donor-patient **pairs**, who are themselves unable to perform a kidney transfer due to incompatibility
- Two or more incompatible donor-patient couples agree that the donors will provide a kidney to another pair's patient with the expectation that their donation will be reciprocated by another pair's donor on behalf of their "own" patient


D1
R1
D2
R2
D1
R1
D2
R2


D1
R1
R3
D2
D3
R2

- Which kidney to match with which patient?
- Form a pool of patient/kidey pairs
- Identify immunologically best fit kidney in the pool to each patient
- Construct a TTC!
- If deaced (or altruistic donors) apply the TTCC extension to form a robust combination of cycles and chains
- Importnat: provides doctors/hospitals with the incentives to enhance exchange


d0
d1,p1
d2,p2
d3,p3
d4,p4
d4,p4
d4,p4
d4,p4
d4,p4
d4,p4
d4,p4

QUESTIONS

- Long transplation chains difficult to excute
- How to optimize the use of deceased donors?
- How frequently should the mechanism be run?
- The larger the pool of patient/donor pairs is, the better functioning the market is

=> **Nordic kidney exchange market!**

THANK YOU!


60˚ 10 1.2 N, 24˚ 57 18 E