Simultaneous Reconstruction of Emission and Attenuation in Passive Gamma Emission Tomography of Spent Nuclear Fuel

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Simultaneous Reconstruction of Emission and Attenuation
in Passive Gamma Emission Tomography of Spent Nuclear Fuel,
to appear in Inverse Problems and Imaging

Outline

The IAEA PGET challenge

Filtered back-projection

Variational regularization for X-ray tomography

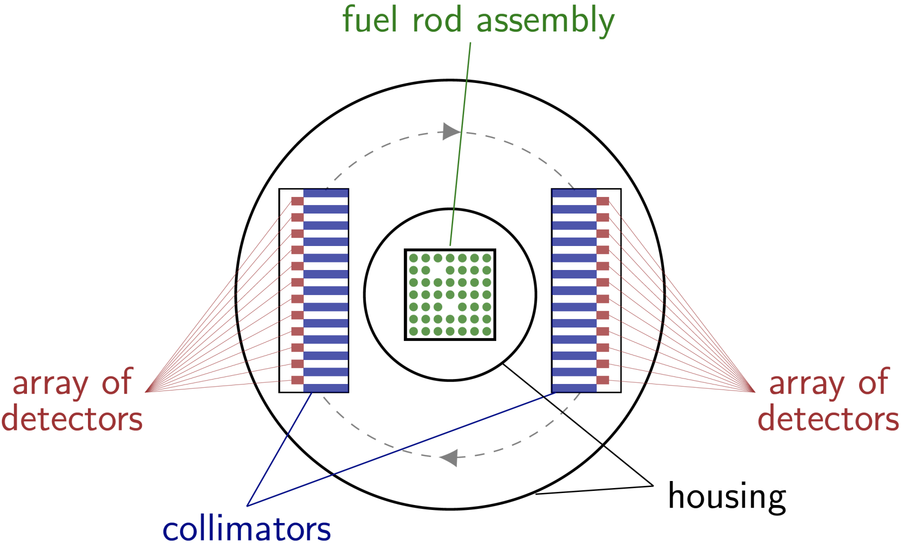
Our PGET method in a nutshell

PGET measurement device

- **P**assive **G**amma **E**mission **T**omography
- Similar idea as in medical SPECT
- PGET strength: ability to image activity of single fuel pins
- IAEA started development in the 80's, approved for inspections in 2017
- Only one device exists at the moment, two more are being built

https://ideas.unite.un.org/iaea-tomography/Page/Home

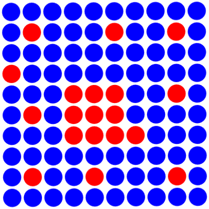
Measurement geometry of the PGET device



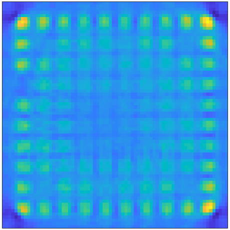
How We Won Silver in IAEA PGET Challenge

Filtered back projection

Ground truth,
present / missing



Activity
reconstruction

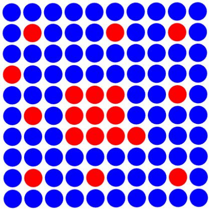


Attenuation
reconstruction

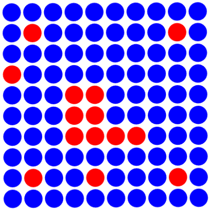
Not applicable

Filtered back projection

Ground truth,
present / missing

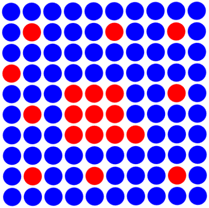


Classification,
present / missing

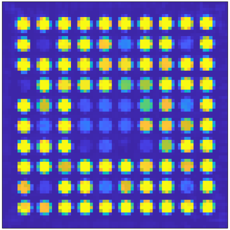


Regularization with geometry aware prior

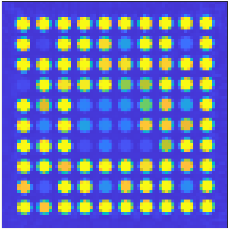
Ground truth,
present / missing



Activity
reconstruction

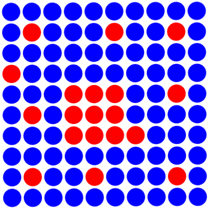


Attenuation
reconstruction

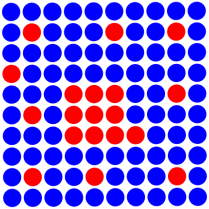


Regularization with geometry aware prior

Ground truth,
present / missing



Classification,
present / missing



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Here is a 2D slice through a human head

Andrew Ciscel, Wikimedia commons

Modern CT scanners look like this

Wikimedia commons

This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAqI

This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58IO1FKw

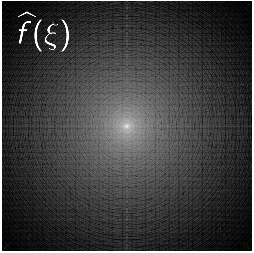
Here is a simple example of tomographic data colle
with two discs as the target

https://youtu.be/5DUGTXd26nA

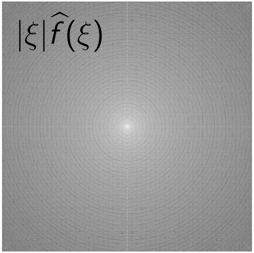
Back-projection twrows the measured data back into the image domain

https://youtu.be/5DUGTXd26nA

Final reconstruction involves filtering on top of the back-projection

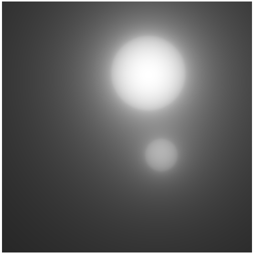


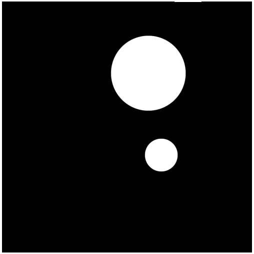
Multiplication with “ice-cream cone”



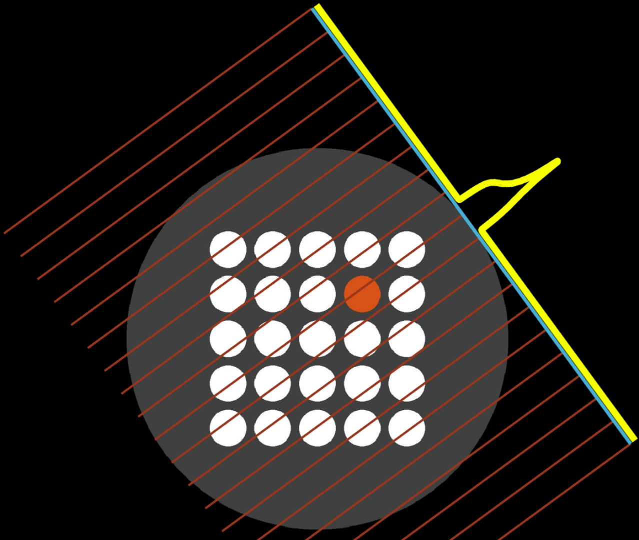
FFT

IFFT





Data is collected by rotating the system around the fuel assembly



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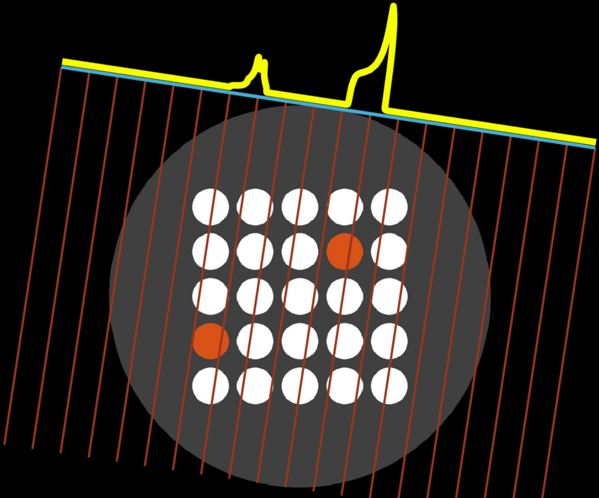
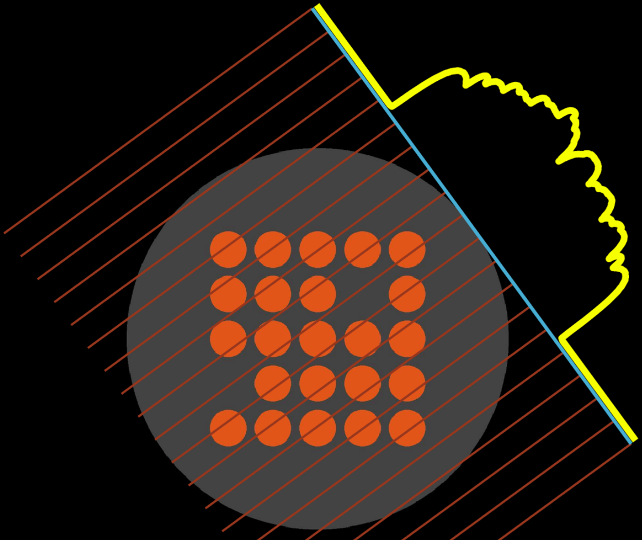


Data is collected by rotating the system around the fuel assembly



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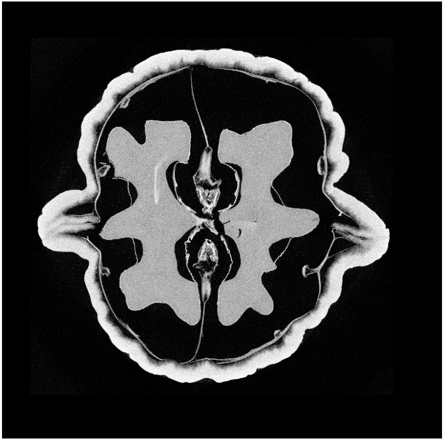
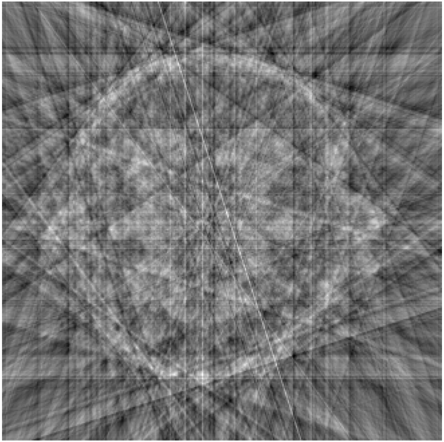
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

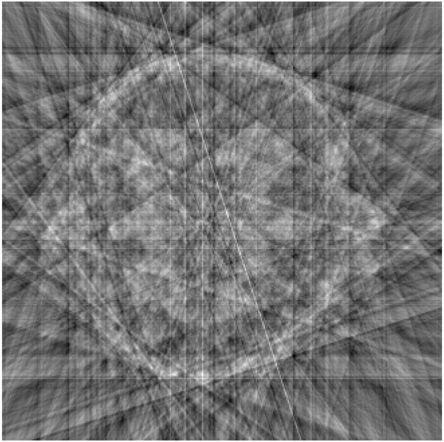


FBP with comprehensive data
(1200 projections)

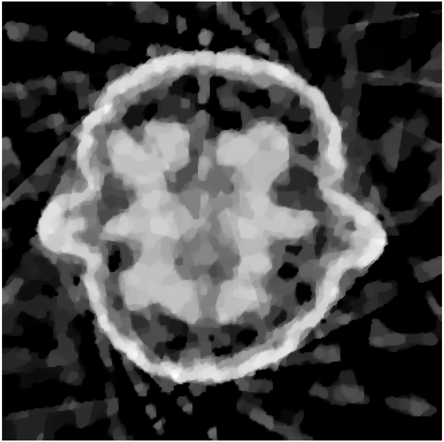


FBP with sparse data
(20 projections)

Sparse-data reconstruction of the walnut using non-negative total variation regularization



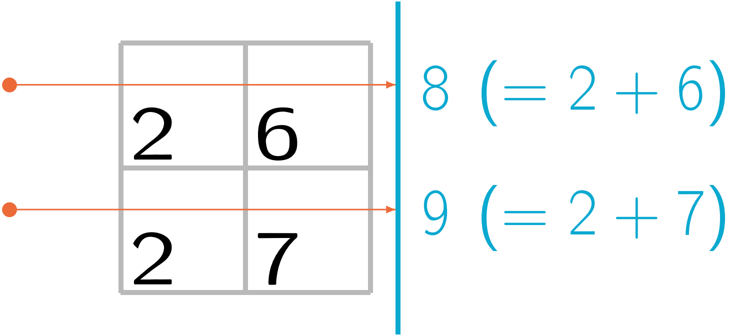
Filtered back-projection



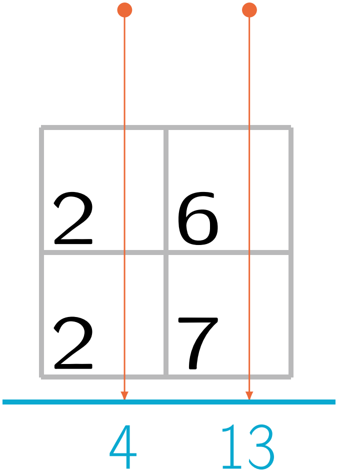
Constrained TV regularization

$$\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}$$

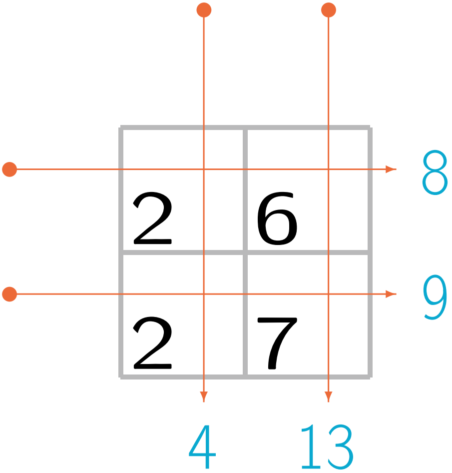
Two horizontal X-rays give us two numbers: row sums of the 2×2 array of attenuations



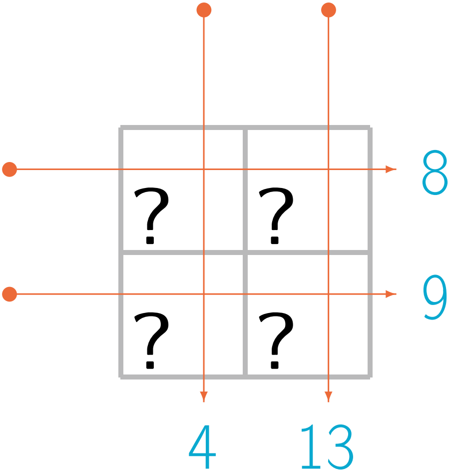
Tomographic imaging requires collecting X-ray data along another direction as well



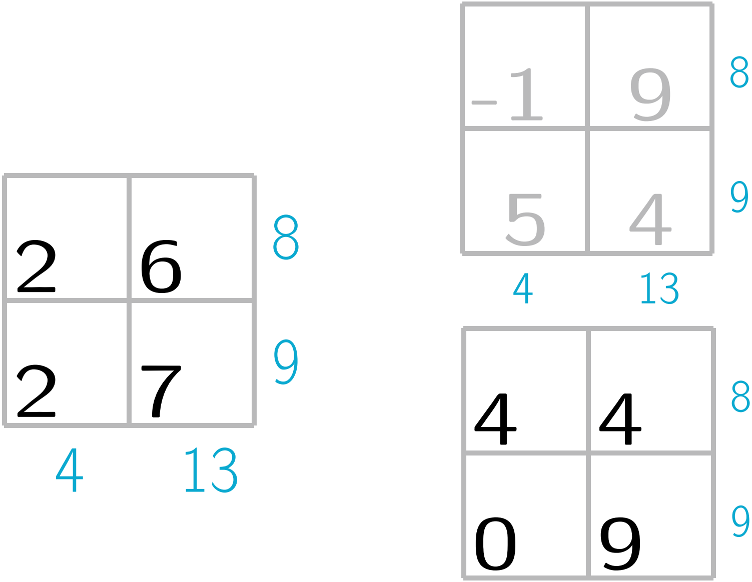
"Direct problem" in this example is to compute row and column sums of a known interior



“Inverse problem” in this example is to recover the interior numbers from the measurements



With such a limited amount of data, the inverse problem has multiple solutions!



Penalty calculation for candidate 1 (true target). First the penalty from (mis)matching X-ray data

2	6	$(8-8)^2$
2	7	$(9-9)^2$
$(4-4)^2$	$(13-13)^2$	

Data penalty: $(8-8)^2+(9-9)^2+(4-4)^2+(13-13)^2=0.$

Penalty calculation for candidate 1.
Then the penalty from prior information

2	6
2	7

Data penalty: $(8-8)^2 + (9-9)^2 + (4-4)^2 + (13-13)^2 = 0.$

Prior penalty: $|2-6|$

Penalty calculation for candidate 1.
Then the penalty from prior information

2	6
2	7

Data penalty: $(8-8)^2 + (9-9)^2 + (4-4)^2 + (13-13)^2 = 0.$

Prior penalty: $|2-6| + |2-7|$

Penalty calculation for candidate 1.
Then the penalty from prior information

2	6
2	7

Data penalty: $(8-8)^2 + (9-9)^2 + (4-4)^2 + (13-13)^2 = 0.$

Prior penalty: $|2-6| + |2-7| + |2-2|$

Penalty calculation for candidate 1.
Then the penalty from prior information

2	6
2	7

Data penalty: $(8-8)^2 + (9-9)^2 + (4-4)^2 + (13-13)^2 = 0.$

Prior penalty: $|2-6| + |2-7| + |2-2| + |6-7| = 4+5+0+1 = 10.$

Penalty calculation for candidate 1. Total penalty is the sum of data and prior penalties

2	6
2	7

	data penalty	0
+	prior penalty	10
=	total penalty	€10

Which of candidates has smallest total penalty?

True target

2	6
2	7

data penalty 0
$+$ prior penalty 10
$=$ total penalty €10

Wrong data,
good "tissue type"

3	3
3	3

data penalty 66
$+$ prior penalty 0
$=$ total penalty €66

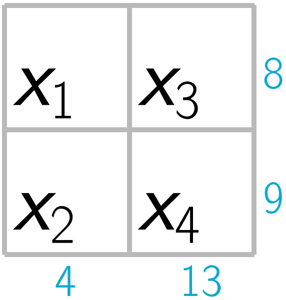
Right data,
bad "tissue type"

4	4
0	9

data penalty 0
$+$ prior penalty 18
$=$ total penalty €18

The problem can be solved in general using optimization

General target



Find numbers $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and $x_4 \geq 0$ such that the sum of these two penalties is as small as possible:

Data penalty: $(x_1 + x_3 - 8)^2 + (x_2 + x_4 - 9)^2$
$+ (x_1 + x_2 - 4)^2 + (x_3 + x_4 - 13)^2$

Prior penalty: $|x_1 - x_3| + |x_2 - x_4|$
$+ |x_1 - x_2| + |x_3 - x_4|$

This method is called **total variation regularization.**

Solution from optimization method

True target

2	6
2	7

data penalty	0
+ prior penalty	10
= total penalty	€10

Total variation regularization

2.5	6
2.5	6

data penalty	1.6
+ prior penalty	7.0
= total penalty	€8.6

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Reconstruction as a minimization problem

Reconstruction images are obtained by solving

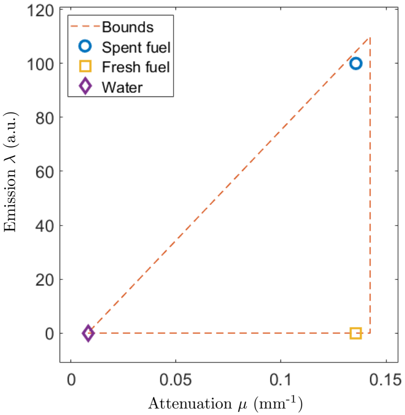
$$\min_{(\lambda,\mu)} \left\{ \|F(\lambda,\mu) - m\|_2^2 + \sum_i \alpha_i P_i(\lambda,\mu) \right\}$$

- λ is the emission image, μ is the attenuation image.
- Data fit term $\|F(\lambda,\mu) - m\|_2^2$ measures how well the forward projection $F(\lambda,\mu)$ matches the measurement m.
- The regularization terms $\sum_i \alpha_i P_i(\lambda,\mu)$ incorporate prior knowledge into the reconstruction process, i.e., they predispose the algorithm towards certain kind of images.

Bounds

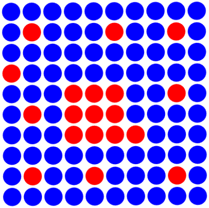
Need to set bounds for the emission and attenuation values in the minimization problem to produce reasonable images.

- Excludes the possibility of a material with high emission but low attenuation value.
- Some way of estimating these bounds before the minimization is needed.

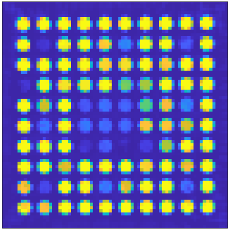


Geometry aware prior

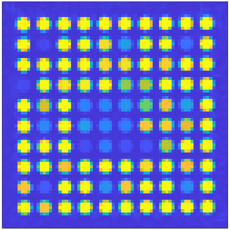
Ground truth,
present / missing



Activity
reconstruction



Attenuation
reconstruction



Thank you for your attention!