Full-core statistical fuel performance analysis

Henri Loukusa

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Outline

- Order statistics
- Calculation chain
- Application to fission gas release
- Application to other fuel performance parameters

Order statistics

YVL guides on statistical methods for deterministic safety analyses

- YVL B.3: “In applying a best estimate method with uncertainty analysis, the result is acceptable if there is a 95% probability with 95% confidence that the examined parameter will not exceed the acceptance limit set for the conservative analysis method.”

Tolerance interval

- Tolerance interval of a parameter defines the probability that a proportion of future samples from the underlying population falls within the interval
- We are concerned with the upper limit, so lower limit can be set to -∞
- Wilks (1942) proved in the 1940s that the order statistics of a sample can be used as nonparametric estimates of the tolerance interval
- Wilks' formula has been widely used in the nuclear industry since the 1980's (for example, in the GRS method, see Glaeser 2008)

$$P\left(\int_{-\infty}^{U} f(x)dx \geq \gamma\right) \geq \beta$$

Wilks, S. The Annals of Mathematical Statistics, 1941, p. 91-96.
Glaeser, H. Science and Technology of Nuclear Installations, 2008, doi: 10.1155/2008/798901

Order statistics

- Order statistics of a sample are the independent observations ordered by their magnitude
- The first order statistic is the smallest observation, nth order statistic is the largest observation

$$X = (x_1, x_2, \ldots, x_n)$$

$$Y = \left(y_1 = x_i, y_2 = x_j, \ldots, y_n = x_k\right)$$

$$x_i < x_j < \ldots < x_k.$$

Tolerance interval from order statistics

- Due to Wilks' proof, it is easy to calculate the number of samples required for a certain order statistic to be greater or equal than the desired tolerance interval
  - In the last step below r = 0 to set L = -∞ and s = N to have the nth order statistic to be greater than or equal to the desired tolerance interval
  - Setting s = N – 1 we would have the (n-1)th order statistic as greater than or equal to the tolerance interval

$$P\left(\int_{-\infty}^{U} f(x)dx \geq \gamma\right) \geq \beta \qquad \beta = I(\gamma; s, N-s+1) \qquad \beta = 1-\gamma^{N}$$

$$I(x;a,b) = \frac{B(x;a,b)}{B(1;a,b)} \qquad B(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}dt$$

Tolerance region

- For multiple output parameters, the tolerance interval of each parameter is not sufficient (Pal & Makai 2002)
- The tolerance region limited by each parameter must be determined
  - More samples from the output distribution are required to satisfy the wanted probability content and confidence level
- The order can be determined by different choices of s_1 below

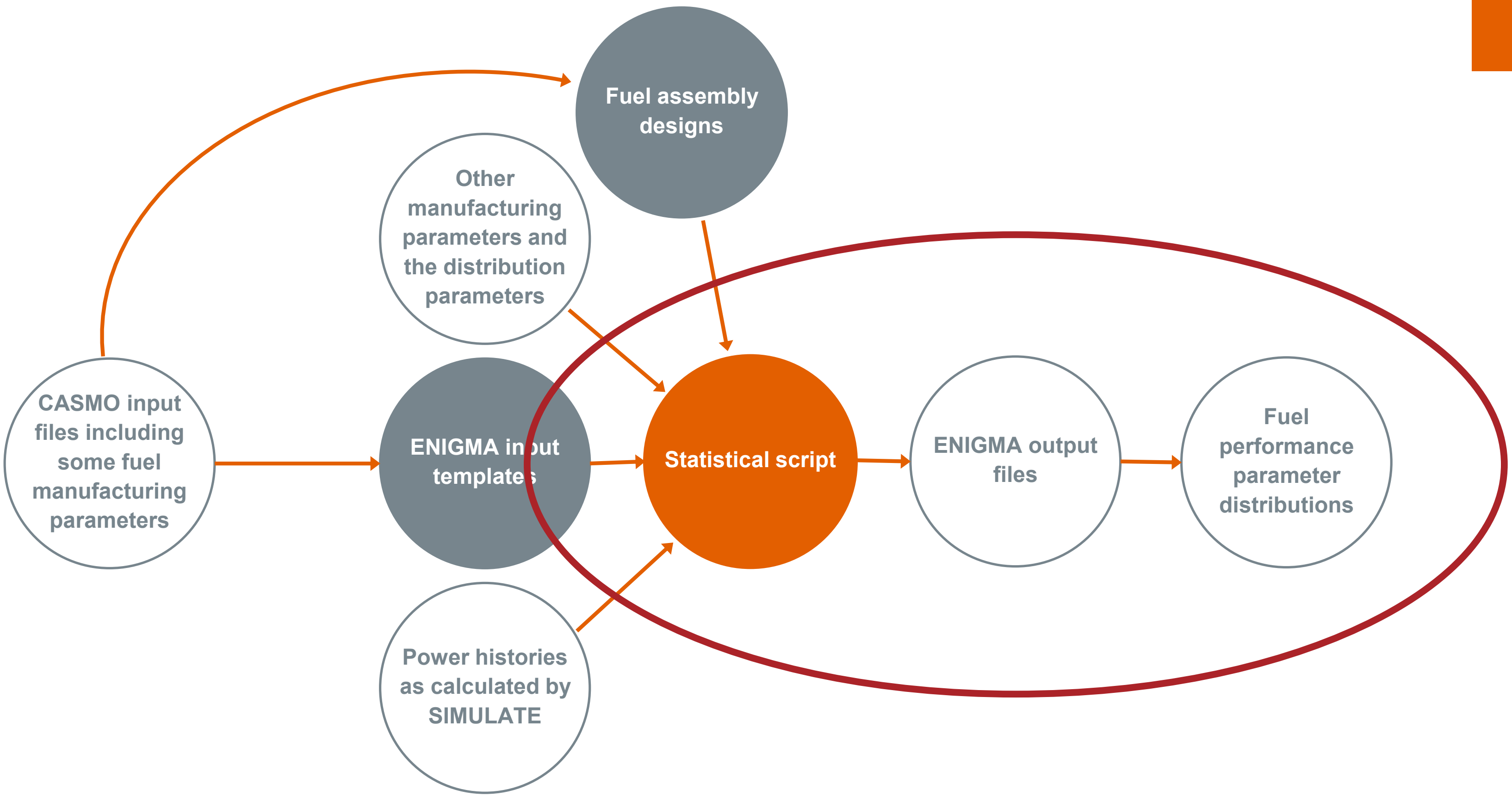
$$P\left(\int_{-\infty}^{U_1} \dots \int_{-\infty}^{U_n} g(x_1, \dots, x_n) dx_1 \dots dx_n \geq \gamma\right) \geq \beta \qquad \beta = I(\gamma; s_1 - n + 1, N - s_1 + n)$$

Pal, L & Makai, M., 2002, arxiv: physics/0308086v2

Numbers of required code runs per rod

		Number of outputs									
		1	2	3	4	5	6	7	8	9	10
Order	1	59	93	124	153	181	208	234	260	286	311
	2	93	124	153	181	208	234	260	286	311	336
	3	124	153	181	208	234	260	286	311	336	361
	4	153	181	208	234	260	286	311	336	361	386
	5	181	208	234	260	286	311	336	361	386	410
	6	208	234	260	286	311	336	361	386	410	434
	7	234	260	286	311	336	361	386	410	434	458
	8	260	286	311	336	361	386	410	434	458	482
	9	286	311	336	361	386	410	434	458	482	506
	10	311	336	361	386	410	434	458	482	506	530

Calculation chain


Fuel assembly designs
Other manufacturing parameters and the distribution parameters
CASMO input files including some fuel manufacturing parameters
ENIGMA input templates
Statistical script
ENIGMA output files
Fuel performance parameter distributions
Power histories as calculated by SIMULATE

Calculation time

- There is a large number of rods in a reactor core and each rod must be calculated several times to yield samples of the output distribution, so the calculation is computationally somewhat intensive
- Rough estimates of calculating the complete core with ENIGMA for each of the Finnish operational and planned reactors are provided in the table below for one output parameter
  - These are extreme values, as typically the complete core does not need to be calculated (due to core symmetry, equilibrium cycle, etc.)
- Calculation is easily parallelized as rods can be calculated independently

Reactor	OL1/2	LO1/2	OL3	FH1
CPU-hours	8300	7700	16000	12400

Application to fission gas release

Acknowledgements to TVO and Posiva for supporting the development of this method

Demonstration case

- Power histories based on nine rods from USPWR 16x16 case in OECD/NEA International Fuel Performance experiments database, automatically generated 236 histories (one assembly)
- Manufacturing parameters from the same source, and fabricated but believable fuel manufacturing parameter distributions
- ENIGMA effective fission gas diffusion coefficient varied
- LHGR varied on each time step

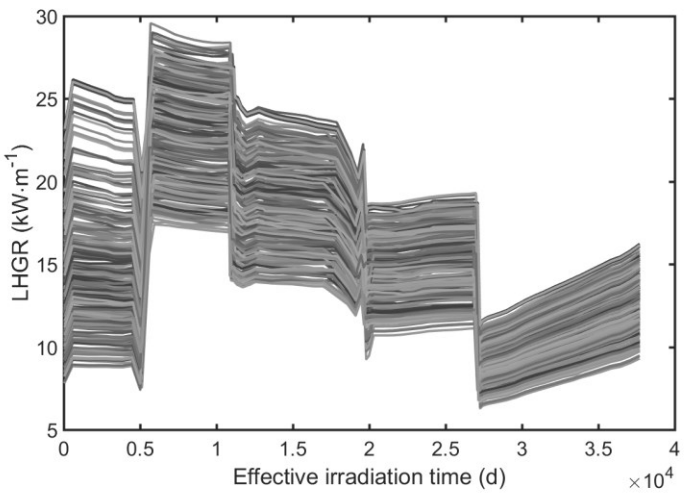
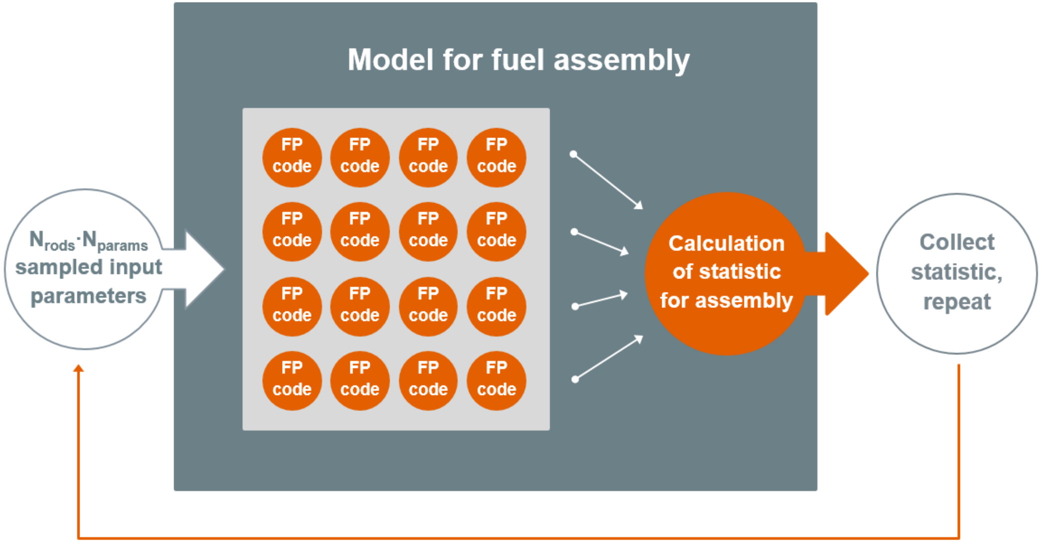


Figure from Loukusa & Tulkki, Nuclear Engineering and Design (paper accepted)

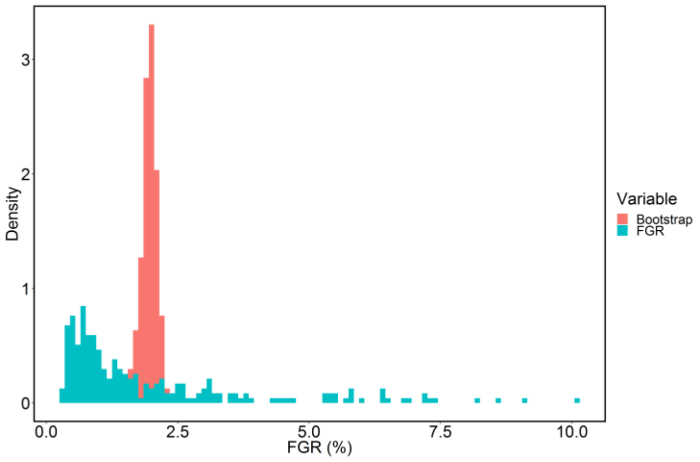
Application to determination of instant release fraction in spent fuel disposal

- The fission gas release (FGR) during fuel operation can be related to the instant release fraction (IRF) of cesium and iodine in spent fuel disposal
- IRF is the fraction of radionuclides that are ”instantly” released on contact with groundwater from a failed canister
  - Thought to be proportional to FGR, and so to follow the distribution of FGR
  - Single rod FGR is not that relevant, but that of a fuel assembly or a whole canister
- Fuel performance analysis treats one rod at a time, so several results from a single assembly (at least) have to be summarized)
- The tolerance interval for the summary statistic can be determined

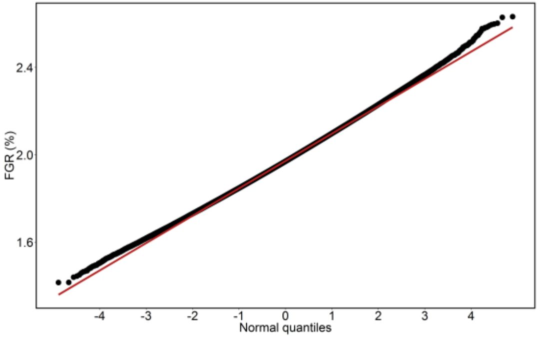
Model for fuel assembly

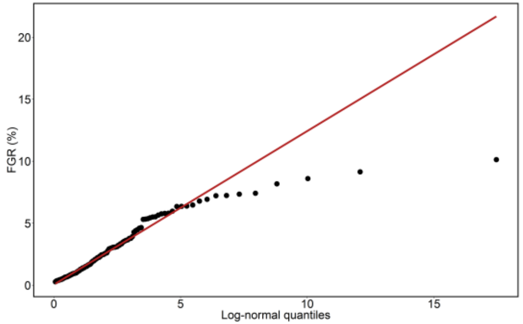


Fission gas release distribution in an assembly



- FGR distribution in the assembly has a long tail
  - ”Cut-off log-normal”
- Distribution of mean FGRs is normal (central limit theorem)
- Tolerance limit determined from mean FGR distribution can be used as a best estimate plus uncertainty value of the assembly FGR





 Figures from Loukusa & Tulkki, Nuclear Engineering and Design (paper accepted)

Application to other fuel performance parameters

Tolerance region method for general fuel behavior analysis

- For fuel in normal operation, the typically important parameters calculated by a fuel performance code are
  - Fuel maximum temperature (no fuel melting requirement in YVL B.4 412.)
  - Rod maximum internal pressure (no lift-off requirement in YVL B.4 412.)
  - Cladding hoop stress (low probability of fuel failure due to pellet-cladding mechanical interaction in YVL B.4 414.)

- All of these requirements must be fulfilled simultaneously, so a tolerance region method must be used
  - Three output parameters with first-order Wilks' formula yields 124 required samples of the joint output distribution (and therefore 124 code runs per fuel rod)

Demonstration case

- Same power histories, model and manufacturing parameter distributions as with the fission gas release demonstration case
- One assembly calculated and for each rod the median and tolerance interval (a dimension of the determined tolerance region) is shown for each studied parameter

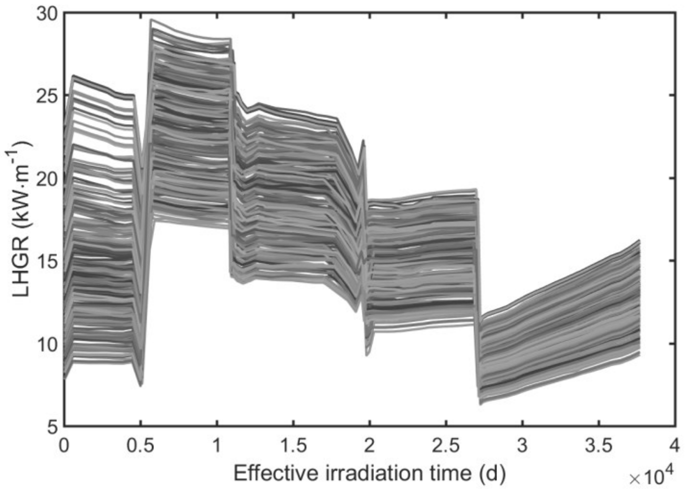
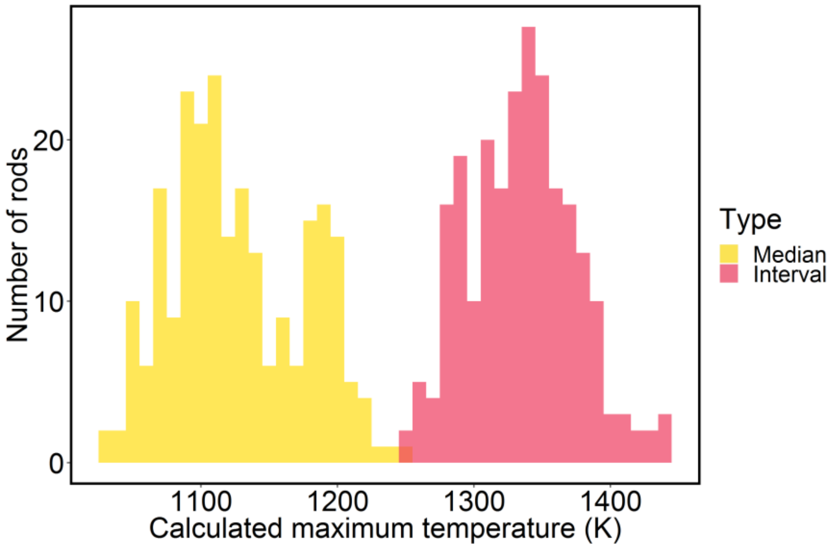
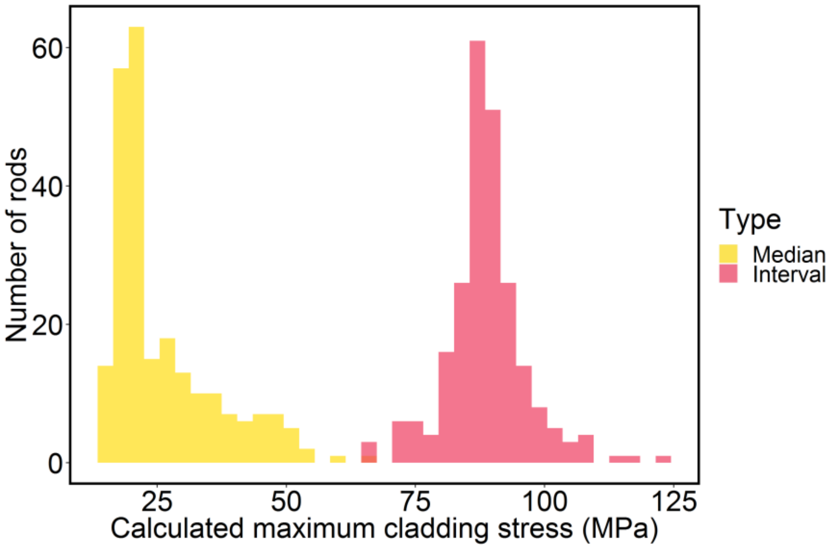


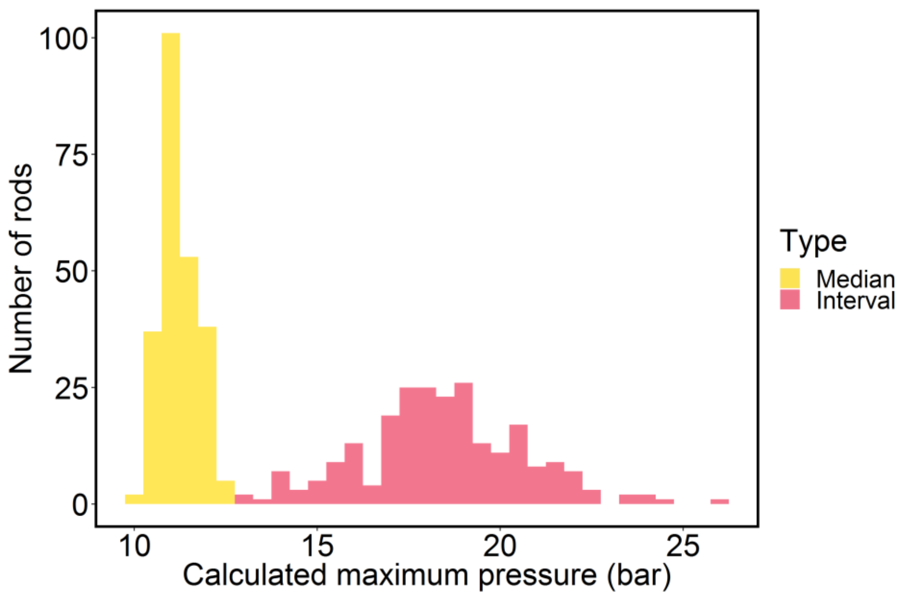
Figure from Loukusa & Tulkki, Nuclear Engineering and Design (paper accepted)

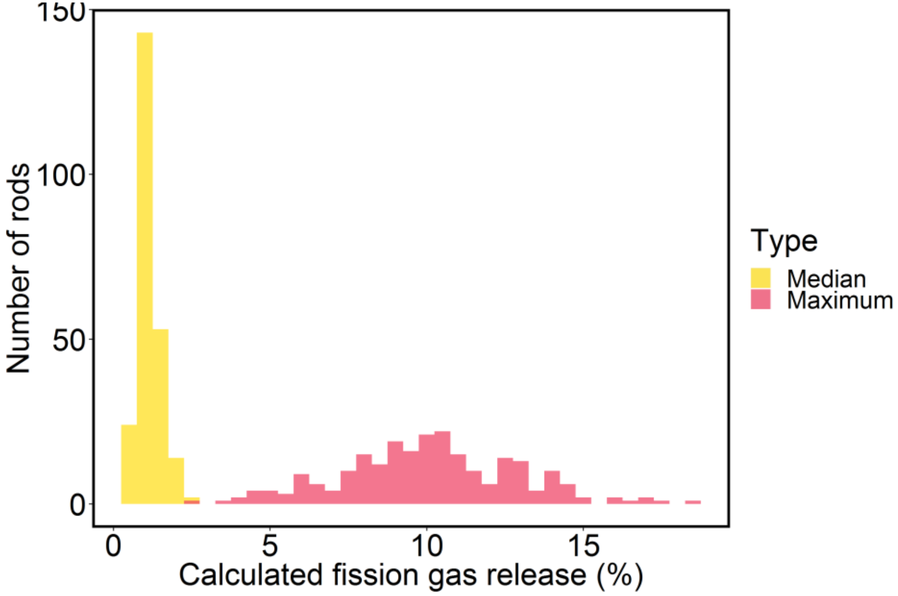
Calculated maximum temperature and maximum cladding hoop stress distributions in an assembly





Calculated rod internal pressure distribution in an assembly (and explanation)





Conclusions

- A method for statistical fuel performance analysis for large numbers of fuel rods has been developed
- The method uses a tolerance interval or region determined with order statistics per the Wilks' formula
- The method was demonstrated with two applications with one and several output parameters
  - Fission gas release relevant to spent fuel disposal safety case
  - Maximum temperature, pressure and cladding stress for fuel in normal operation

beyond
the obvious

Henri Loukusa
henri.loukusa@vtt.fi
+358 40 8258286

www.vtt.fi

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